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A TEXT-BOOK

ON

STATIC ELECTRICITY

BY

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PREFACE.

The author was led to the preparation of this volume by the apparent lack of any suitable work of similar description. The subject of Static Electricity is touched on in the average "Physics" or "Natural Philosophy" in a most gingerly fashion; text-books devoted entirely to electricity seem to be either "Electric Machines" or "Alternating Currents," and avoid the subject entirely save for some slight consideration of the electrodynamic aspects of the condenser; some of the very numerous "Electricity and Magnetisms" have a certain amount of matter, not happily arranged for either a text-book or a book of reference, and are often unfortunately loose in statement and definition; and it would seem that the only adequate literature on the subject lay in papers so profound as to be practically inaccessible to the ordinary student or to him who desires to make a reference without reading whole volumes.

And so the author, who has had perforce to do a certain amount of reading on the subject, ventures to put forth this work. There is little or nothing original in it, but it is hoped that by the judicious combining of the wisdom of others in lucid and orderly fashion, and by keeping strictly to the subject of Electrostatics, he has made a volume valuable to the student, the physicist, and the casual seeker after electrostatic truths.

There has been a constant endeavor throughout the work to make the reading as simple as is consistent with exact-

ness and a reasonable terseness. The methods of the calculus have been avoided save where demanded for preciseness or compactness of expression. The copious index will enhance its value as a book of reference.

In the Appendix have been put two tables and a few paragraphs which, while not strictly appertaining to the subject of this book, will, it is believed, be of interest to him who is interested in electrostatics. Many students of electricity are hazy regarding, if not quite ignorant of these two subjects, the physical dimensions of units, and the relation between the units of the electrostatic series and those of the electromagnetic system.

Acknowledgment of indebtedness is herewith made to those manufacturing concerns that have given information regarding, and loaned electrotypes of their apparatus.

HOBART MASON.

BROOKLYN, N. Y., *November,* 1903.

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STATIC ELECTRICITY.

CHAPTER I.

GENERAL PHENOMENA.

1. Historical.—Electricity is the name given to that which produces electrical phenomena, but while electrical phenomena are well known and understood, the nature of electricity is as yet unknown. The term "electricity" comes from the Greek ἤλεκτρον, amber, since, as Thales of Miletus (600 B. C.) informs us, the ancient Greeks recognized the fact that amber when rubbed attracted light particles to it, or as we would say, became electrified. Until about 1600 amber and jet were the only substances that were known to have this property. Then Dr. Gilbert discovered that many substances, such as glass, resin, sulphur, and others, also possessed this property, so the seventeenth century marks the real beginning of electrical science.

2. Attraction.—The most obvious property of an electrified body is that of attraction. This property is easily illustrated by rubbing a piece of amber, as did the Greeks, and noting that it will then attract dust, chaff, small bits of straw and paper, or a pith ball suspended on a thread. In experiments of this nature it is usual to use not amber for the body to be electrified, but a stick of vulcanite (hard rubber) which is rubbed by a

piece of cat's fur, or a glass rod rubbed by a silk cloth. The property of attraction can further be well shown by holding the electrified stick of vulcanite near a thin stream of water, as in Fig. 1, and noticing that stream is deflected so as to flow nearer the vulcanite. Since the presence of moisture precludes the possibility of successful electrification, care must be taken in this experiment that neither the stream nor any spray from it be allowed to touch the electrified body.

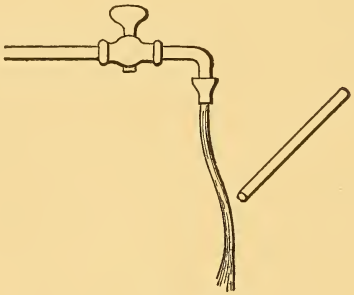

FIG. 1.

Electrostatic attraction can also be shown by briskly drawing a sheet of paper between the coat-sleeve and the side. The paper is then electrified, and if laid against the wall without more handling than necessary, it will so strongly attract the wall as to sustain its own weight until its electrification is exhausted, perhaps for a quarter of an hour. Like all other electrostatic experiments, this one will work more satisfactorily on a dry day, and even a reasonable amount of humidity may cause it to fail entirely.

Those who have dry, light hair have often observed that, on running the comb through it, the hair is attracted

by the comb even after the latter has been removed some inches from the head. This happens only on dry days and with combs of hard rubber or similar material. Metal combs cannot be made to produce this effect. It sometimes occurs that on brushing the clothes it seems impossible to remove the dust, and the more vigorously the brush is wielded the more tenaciously the dust seems to stick. This is due to the electrification of the cloth and the consequent attraction of the dust particles.

3. Electrification and Charge.— When a body has acquired the power of attraction illustrated in the last section it is said to be electrified, or to carry a charge of electricity. In the consideration of this matter it is convenient to think of the electricity as something material that is laid on to the body in charging it. The quantity laid on is a measure of the *quantity* of the charge, and the thickness with which it covers the body is a measure of the *surface density,* sometimes called *tension,* of the charge. It must be remembered that this idea is entirely fictitious, but it affords a convenient way of gaining a comprehension of the actions and relations of electricity which in itself is uncomprehended.

When a charged stick of vulcanite attracts to itself a piece of paper, energy must be expended in moving that paper. Since the vulcanite was powerless to move the paper before it was electrified it must have received the energy in the process of charging, since by the law of conservation of energy, energy cannot be created, but only transferred and transformed. If the vulcanite had been charged by rubbing with cat's fur, enough energy was expended in the rubbing to account for the energy the vulcanite afterward showed itself to possess when it attracted the paper chip. If the vulcanite had been electri-

fied or charged by contact with another charged body, it would have received its energy along with its charge from that body and the sum of energy in the two bodies would then be equal to the energy the original charged body had at first. Thus it is seen that when a body is electrified or charged it possesses an increased energy, and that energy must have been expended to bring it to the state of electrification.

4. Repulsion.—Attraction is not the only property of a charged body. If a pith ball suspended on a thread be attracted by an electrified glass rod, and be allowed to touch that rod, it will immediately be repelled and held at a distance. The reason is that the pith ball on touching the glass received some of the charge from the glass, and bodies similarly charged repel one another.

This repulsion of similarly electrified bodies is shown by bringing a charged ebonite rod near a jet from which a small stream of water is issuing. Through the *influence* (§ 9) of the charged rod the water particles become themselves charged each with a similar charge, and therefore repel each other and the stream breaks up into a spray. In a capillary tube, through which water will not ordinarily flow, the fluid can sometimes be made to pass by the presence of an electrified body in the neighborhood. The particles are then similarly charged and have a tendency to repel each other. As a consequence the surface tension of the liquid is lessened and the water will flow. Similarly a fine stream of water may be made to issue from a pin hole in the bottom of a charged containing vessel through which it will not pass when not electrified.

All electrified bodies do not repel one another, but only those that are similarly electrified. To show this, let a vulcanite rod rubbed by a woolen cloth be approached to

a suspended pith ball. The latter will be attracted until it has touched the rod and received some of the charge, when it will be repelled. Now let a glass rod rubbed by a silk cloth be approached to the pith ball and it will be very strongly attracted. In fact the attraction may be noticed to be stronger than it would have been had the ball not been previously charged by contact with the vulcanite rod. Let, however, the ball touch the glass rod

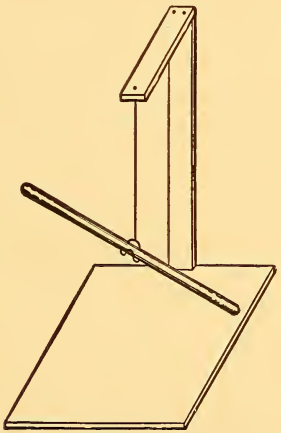

FIG. 2.

and it gives up its previous charge and takes on some of that of the glass, and is, in consequence, repelled. The pith ball is now strongly attracted by the vulcanite rod. This experiment shows that while bodies similarly charged repel one another, two bodies charged by different means may attract each other.

5. Positive and Negative Charges.— In order to investigate the different kinds of electrification, let a glass rod be swung in a stirrup by a thread, as in Fig. 2. Charge

the rod by rubbing briskly with a silk cloth. If now another glass rod, also charged by rubbing with a silk cloth, be approached to the suspended rod, a repulsion is observed. Let, however, a rod of vulcanite, electrified by rubbing with a woolen cloth, be approached, and an attraction is observed. Substituting in turn sulphur, metals, resin, and celluloid, each rubbed by silk, and attraction is observed. Clearly then, there must be two classes of electrification, one attracting the charge on the suspended rod, the other repelling it. In former years, on the supposition that glass could have but the one kind of electrification, the two kinds of electrification were called respectively *vitreous* and *resinous*. The unsuitability of these terms is shown by the following experiment. Let the glass rod in the stirrup (Fig. 2) be rubbed by a silk cloth and another glass rod be rubbed by a woolen cloth or a piece of fur. On approaching the second to the first an *attraction* is observed. From this we see that the kind of electrification resulting from the rubbing of a substance depends not only on that substance, but also on the material with which it is rubbed. To distinguish the two kinds of electrification, it has been arbitrarily decided that the charge on a glass rod resulting from rubbing it with a silk cloth shall be called a positive charge; that all charged bodies which this glass rod tends to repel are also positively charged, and that those charged bodies which it tends to attract are negatively charged.

It is further true that neither kind of charge can be produced alone. When a vulcanite rod is electrified by rubbing with a woolen cloth, the cloth receives a positive charge exactly corresponding to the negative charge on the rod. That the cloth is charged may be shown by some of the means employed in the preceding experiments,

or more easily by means of the electroscope, an apparatus described at the end of this chapter.

6. Electrostatic Series.—By continued experimenting, rubbing different substances together in pairs, and noting which is positively and which negatively charged, the following condition is found to exist. If *A* be charged positively when rubbed by *B*, and if *B* be charged positively when rubbed by *C*, then *A* will be charged positively when rubbed by *C*. This condition makes it possible to arrange a series of substances in such order that each, when rubbed by any one preceding it in the list, becomes negatively charged, and when rubbed by any one succeeding it in the list becomes positively charged. The following series is so arranged that if any two of the substances mentioned be rubbed together, that one which stands earlier in the list carries the positive charge, the other the negative:

Fur,	The hand,
Wool,	Wood,
Ivory,	Shellac,
Feathers,	Metals,
Quartz,	Resin,
Glass,	Sulphur,
Cotton,	India rubber,
Linen,	Gutta-percha,
Silk,	Celluloid.

Electrification occurs whenever two bodies not exactly similar in both chemical constitution and molecular arrangement are in contact. The condition of the surface of the substances exercises an influence over their electrical relations. For this reason plain and ground glass may be made to show slight electrical differences.

7. Volta's Series.—It is generally believed that it is not the friction of rubbing that causes electrification, but the increased intimacy of contact that is caused by the motion. In other words, electrification is due to contact of dissimilar substances. Volta showed that contact of different metals produced electrification by using a *condensing electroscope,* an apparatus for detecting very faint charges, which is described in section 27. He arranged a series, similar to the electrostatic series, but of the metals only. As before, each metal is positively electrified by contact with any of those following it in the list:

Zinc,	Copper,
Lead,	Silver,
Tin,	Gold.
Iron,	

More recent investigations have put sodium and magnesium at the head of the list and platinum and carbon (graphite) at the end.

It must be borne in mind that this series is only true for contact of metals in *air,* since in other atmospheres a different order is needed. For instance, in air iron is positive to copper, yet in an atmosphere of hydrogen sulphid iron is negative to copper.

8. Conductors and Insulators.—While all bodies can be electrified, only certain of them offer indications of electrification in the way of attracting pith balls or repelling similarly electrified bodies. The reason is that in some substances the charge remains where generated, while in others it disseminates to all parts of the body immediately and has not sufficient intensity to give such indications. Substances which allow of the dissemination

of the charge are called *conductors,* while those on which the charge remains where generated are called *insulators* or *non-conductors.* These conditions are relative, not absolute. Both copper and iron are called conductors, yet copper is from 6 to 8 times as good a conductor as iron. Dry wood is an insulator to the current from an ordinary galvanic cell, but is a conductor to the higher tensions of electrostatic phenomena, and cannot replace the vulcanite or glass rods in the various experiments already described. Pure water is a very good insulator, but ordinary water, holding CO_2 in solution, is a rather good conductor. The following list of substances, according to Prof. S. P. Thompson, is arranged in the descending order of their conductivities:

Silver,	Porcelain,
Copper,	Wool,
Other metals,	Silk,
Carbon,	Resin,
Water,	Gutta-percha,
The body,	Shellac,
Cotton,	Ebonite,
Dry wood,	Paraffin,
Marble,	Glass,
Paper,	Fused quartz.
Oils,	Dry air.

In considering the choice of a conductor or an insulator for any given purpose, not only must its position in this series be regarded, but its mechanical and chemical properties must be taken into account as well. For conductors copper and iron are the two substances most generally used. Silver is a better conductor than either, but is too

expensive for practical use. Copper is more expensive than iron but is so much better a conductor that its extra cost is more than compensated for. Iron, besides being cheaper, has the additional advantage of greater tensile strength, which is an important advantage where wires are strung on poles. Carbon is used as a conductor in both incandescent and arc lamps, and mercury, the best known fluid conductor, is, because of its convenience, much used in electrical laboratory instruments.

The proper choice of an insulator for any particular purpose is not a simple matter. Hard rubber, ebonite, or vulcanite, is much used and has many advantages in point of strength and ease of working. But it is open to the serious objection that it looses its insulating quality when exposed to light. This is because the sulphur used in the process of vulcanizing oxidizes in the presence of light. The sulphur oxid then takes up water from the atmosphere, forming sulphuric and sulphurous acids. These acids, which may be easily tasted by applying the tongue to a piece of the rubber, are good conductors, and hence as an insulator the hard rubber defeats its own purpose. The brown, dull appearance of a piece of hard rubber that originally was black and highly polished, is due to the destroying action of these acids. A partial remedy for this trouble is had by washing the surface well with a weak alkali solution, ammonia, for instance, then with clear water, and finally rubbing it well with paraffin oil. This does not entirely obviate the trouble, since the oxidization occurs to some extent beneath the surface, where the alkali wash does not reach. Another very serious objection to hard rubber is its extraordinarily high coefficient of expansion, which is more than twice that of glass. This fact alone prevents its use in many

cases where the different parts of an instrument must maintain their same relative positions through a considerable range of temperatures.

Glass is widely used as an insulator. Its chief objections are its brittleness and the difficulty of working it. It is also very easily wet by water, hence a single drop of moisture will spread over a large surface of the glass and materially increase the "surface leakage."

Quartz, next to dry air, is the best of insulators. It has no elastic fatigue, and this quality makes it the very best substance where an insulating, flexible, supporting fiber is desired. Furthermore it does not wet all over, but what moisture settles on it collects into little globules like mercury on a dusty table. The chief objection to it is the great difficulty of working it and the consequent expense.

Paraffin, called ozokerite, is a good soft insulator, but it collects dust, the dust in turn absorbs moisture from the atmosphere and surface leakage then occurs. Paraffin also has a large coefficient of expansion, but in so plastic a substance, this of little importance.

Mica is a good insulator and cannot be excelled where a thin, strong, and rigid layer of insulation is required. Its peculiar structure, however, prohibits its use in almost all other cases. Mica scales or chips are mixed with oil, shellac, rubber, or other materials, making somewhat flexible insulating sheets sold under various trade names.

Kerosene oil is a good insulator, but is carbonized by the passage of an electric spark or by the "silent" or "brush" discharge, and when carbonized its insulating qualities are impaired. Animal and vegetable oils are apt to contain organic acids which are more or less conductors, and in any case such oils are liable to decomposition which would render them conductors.

Shellac is a good insulator when properly prepared. It should be dissolved in good alcohol and kept from accumulating dirt while in its liquid state. Shellac made up with wood alcohol contains an organic acid which not only impairs its insulation but attacks many metals with which it may come in contact.

9. Influence.—A body may become charged without either rubbing or contact with another charged body,

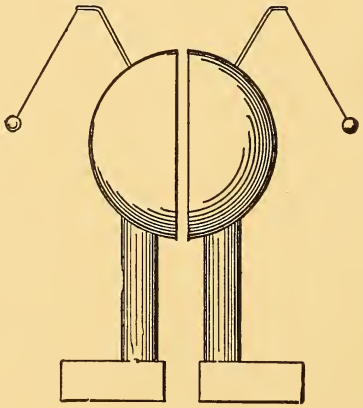

Fig. 3.

through the *influence* of another charged body in its neighborhood. This is shown in the following experiment. Let the two similar metal hemispheres, Fig. 3, supported on insulating stands, be placed with their faces in contact. Each hemisphere has a short wire projecting from it, from which is suspended by an insulating thread a small pith ball. Now let a vulcanite rod be briskly rubbed with a woolen cloth and brought near the composite sphere. We now find that the point of the sphere nearest the rod, and

the point farthest from it have the power of attracting small bits of paper. We see also that the pith balls are repelled. Now by testing with a body of a known electrification we find the ball on the side of the sphere nearest the vulcanite rod is positively charged, while that on the far side of the sphere is negatively charged. Now since they are repelled from the sphere, the latter must have a positive charge on the near side and a negative charge on the far side. This is evidently due to the *influence* of the vulcanite rod, for the whole phenomena ceases on its removal. To clearly understand this phenomenon it is convenient to adopt the " two fluid " theory, according to which every material body contains equal amounts of positive and of negative electricity, and on electrifying such a body the fluids are separated, the preponderance of one or the other fluid giving rise to the phenomena which are attributed to charge. Thus on rubbing the vulcanite rod with the woolen cloth the two electricities in each of them were separated, the negative collecting on the rod, while the positive passed over to the cloth. Now, when the rod was brought near the sphere, the two electricities of the latter were separated, the negative electricity of the rod attracting the positive electricity of the sphere while it repelled the negative of the sphere, hence two charges rested on the sphere, a positive charge near the rod, a negative one opposite it. This action it will be observed does not draw from the charge on the rod, and the two " fluids " will mix immediately on the removal of the rod. If now, while the rod is still near, the farther hemisphere be removed from its mate there will be a surplus of negative electricity on it which cannot be neutralized, when the rod is removed, by the positive " fluid " on the other hemisphere because of the insulating

air between. Therefore each hemisphere will carry a charge although they are good conductors and could not be successfully electrified by rubbing. This phenomenon of *influence* is sometimes called *induction.* This is unfortunate, as *induction* is more widely used as meaning the generation of electromotive force in a conductor cutting magnetic lines of force; and to distinguish the two, the unwieldly expressions electromagnetic induction and electrostatic induction must be used. So throughout this work *influence* and *influenced* will be used to the exclusion of *electrostatic induction* and *electrostatically induced.*

10. Bound and Free Charges.— If in a conductor a charge is held at some particular point through the influence of a neighboring charge, the charge so held is said to be *bound.* For every bound charge on a conductor there must be an equal charge of opposite kind in a remote part of that same conductor or a conductor connected with it (provided the continuity of such conductors is not disturbed during the experiment and that they were not already carrying free charges). If there is an actual surplus of one kind of electricity on a conductor and if it is maintained on that conductor simply because the latter is completely insulated and the charge has no conducting path to lead it away, then such charge is said to be *free.* There can be no free charges of opposite sign in the same conductor at the same time. For if there were two such charges they would immediately flow together. If they were equal they would completely neutralize each other, leaving the conductor uncharged, and if they were unequal the smaller would neutralize an equal part of the larger, leaving the conductor with a diminished charge of the same sign as the larger. For example, in the experiment cited in the last section, where the hemispheres were

together and the charged vulcanite rod was approached, a positive charge was bound on the nearer side of the sphere. When the hemispheres were separated and the rod removed, the charges distributed themselves over their respective conductors and the nearer hemisphere held a free positive charge and the farther a free negative charge.

Of course a bound charge may be superposed on a conductor already carrying a free charge. In this case the free charge is attached or repelled according to its sign, and adds itself to the bound charge or to its opposite which is located at some remote part of the conductor. In this case the bound charge does not have an equal and opposite charge in the same conductor, the difference being just the amount of the original free charge.

A single free charge can be put on a non-separable sphere by proceeding thus: First approach the charged vulcanite rod, then touch the sphere with the fingers. The negative "fluid" of the sphere in trying to get as far as possible from that on the rod will flow through the body into the ground. It is quite immaterial what part of the sphere is touched. Even if the fingers be placed on that part nearest the rod, the positive charge will still be held bound and the negative repelled to the earth.

Now remove the fingers to prevent the return of the negative electricity and then remove the rod and a surplus of positive electricity is on the sphere which cannot disperse or neutralize itself because surrounded by insulators. In other words, the sphere carries a free positive charge.

An application of this principle is found in the electrophorus, illustrated in Fig. 4. A circular disc of hard rubber, sulphur, or a mixture of shellac, resin, and Venice turpentine is suitably held in a metal dish. A metal

"cover" fitted with a glass handle, and of a somewhat smaller diameter than the disc, rests upon the latter. If now the disc be briskly rubbed with wool, or better with cat's fur, it becomes negatively electrified. Now placing the cover on the disc, by influence a positive charge is produced on its under side and a negative charge on top. On touching the upper side with the hand the negative "fluid" flows to the ground. On removing the hand there is a quantity of positive electricity bound on the under side

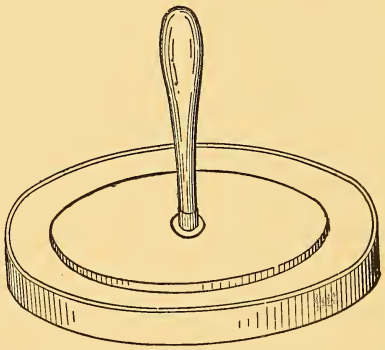

FIG. 4.

of the cover that becomes a free charge when this conductor is lifted by the insulating handle. Now on approaching the cover to the knuckle or other conductor connected to the ground, the positive charge attracts a corresponding quantity of negative electricity to the knuckle. When the distance is sufficiently reduced the attraction between these two charges becomes so great that the electricity jumps through the air in the form of a spark and a redistribution of the fluids takes place such that the cover is once more neutral or, as it is called, discharged. The cover

may be laid on the disc again and the whole operation repeated a number of times until the charge on the disc becomes too weak because of leakage, as it eventually will, since neither the air nor substance of the disc is a perfect insulator.

11. Electroscopes.—Any instrument for detecting the presence of electrification and determining the kind of electrification is called an electroscope. Hence the pith ball on a string is a crude form of electroscope. A somewhat better instrument is made by mounting a long thin

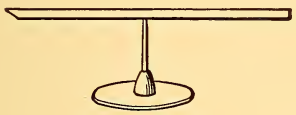

FIG. 5.

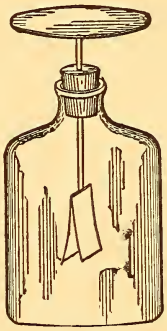

FIG. 6.

and very light rod of almost any material, wood, straw, brass or other, on a needle point so as to swing freely in a horizontal plane like a compass needle, as in Fig. 5. This will show the presence of charges far too faint to affect a pith ball.

The *gold-leaf electroscope* is, however, the most delicate and most used of this class of instruments. As seen in Fig. 6, a flask or other suitably shaped bottle has its mouth corked and the cork penetrated by a glass tube. Through this tube runs a brass or copper rod connected to a brass

disc above and below to two similar pieces of gold-leaf hanging side by side and united at their upper edges only. On bringing a charged body in the vicinity of the disc the leaves will diverge, since through the effect of influence, a charge will rest on the disc and a charge of the opposite kind will rest as far from it as possible in the same conductor, or in the gold leaves, and each of the leaves having a similar charge, they will be mutually repelled. The following is the procedure to determine the kind of electrification that a charged body carries. Let a body be charged with a known kind of electricity, say by rubbing a rubber rod by a woolen cloth giving a negative charge on the rod. Approach this rod to the disc of the electroscope. A positive charge will be influenced on the disc and a negative charge in the leaves. Now touch the disc with the hand. The negative fluid will flow to the ground. (§ 10.) Remove first the hand and then the rod. The leaves will diverge less than at first, since the negative charge existed in them alone while now an equal positive charge is spread over the whole of the conductor. The leaves will, however, be separated quite sufficiently. Now bring the body whose electrification it is desired to test into the vicinity of the disc. If the leaves diverge further, the body is positively electrified, if they come together some, the body is negatively electrified. For if the body be positively charged, this charge will repel the positive "fluid" in the conductor of the electroscope to the leaves, thus increasing the intensity of the charge there and consequently increasing the divergence. If, however, the body be negatively charged, this charge will attract the positive "fluid" of the conductor to the disc, thus reducing the intensity of the charge in the leaves, and consequently reducing the divergence. A free *negative* charge could be placed on

the conductor of the electroscope by touching the disc with the negatively charged rubber rod. It is better, however, to charge the instrument by influence as indicated, as in this way the amount of charge can be regulated, while if a highly charged body be touched directly to the disc, the gold leaves may repel each other so violently as to do themselves damage.

CHAPTER II.

THE ELECTROSTATIC FIELD.

12. Coulomb's Law.—Coulomb's law is stated as follows: The force between two charged bodies, far apart as compared with their dimensions, is proportional to the product of the charges, and inversely proportional to the square of the distance between them. Or $F = \frac{qq'}{r^2}$ where q and q' are the magnitudes of the charges on the two bodies respectively, and r is the distance between them.

This law deduces directly from experiments. The apparatus used by Coulomb is known as the torsion balance. A pith ball is mounted on a light insulating horizontal arm. At the center of gravity of the combination is attached a vertical supporting fiber of glass or metal. The pith ball is charged, and a similarly charged body is brought into proximity therewith. The ball is repelled, and is maintained at a suitable position — far when compared to its own diameter — by twisting the supporting fiber. The torque, and hence the force between the charged bodies, is measured by the angle through which the fiber is twisted. Now, to subdivide one of these charges, let the pith ball be placed in contact for a moment with an exactly equal and similar ball which is electrically neutral, *i. e.* without any charge. The charge previously on the first becomes evenly distributed over both, and on removing the second, the first is left with just half the charge it held at first. To cause the balance to maintain the same position as before, it is found that just half the

twist on the fiber is required as was in the first instance. By subdividing either or both of the charges to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc., it is found invariably that the force between them is directly as their product. By maintaining the charges constant but varying the distance between them, it can be shown that the force varies inversely as the square of the distance.

These experiments should be carried out either in dry air or in vacuo, with no other charges and no electrical conductors in the neighborhood.

13. Quantity.— From the expression $F = \frac{qq'}{r^2}$ may be derived the electrostatic unit of quantity of electricity. The units of F and r are already known, the dyne and the centimeter. To get unit conditions let $F = 1$ and $r = 1$ and $q = q'$, then must $q = 1$. In other words: The unit quantity of electricity is such a quantity that when placed in air one centimeter from an equal and similar quantity of electricity, it will repel it with a force of one dyne.

It is only in this definition of quantity that the definitions of the absolute electrostatic units and of the absolute electromagnetic units do not agree. The electromagnetic unit of quantity is defined as that quantity which when flowing in one second through a conductor one centimeter long, bent in the arc of a circle of one centimeter radius, exerts a force of one dyne upon a unit magnet pole situated at the center. This electromagnetic unit is about thirty thousand million times the electrostatic unit. Since all the other definitions in the two systems are practically alike in wording, and since they all include the unit of quantity, it follows that the constant numerical factor that connects similar units of the two systems is thirty thousand million, or more exactly 2.9857×10^{10}. By repeated

observations it has been found that the velocity of propagation of light in air is 2.9992×10^{10} centimeters per second, and the striking similarity of these figures is of considerable significance in sustaining the ether theory of electricity.

The practical electromagnetic unit of quantity is the coulomb, and is one-tenth of the absolute unit. There is no corresponding practical unit in the electrostatic nomenclature, nor is any special name given to the absolute unit. For further comparison of these units, refer to the Appendix.

14. Surface Density.—As before stated, it is convenient to consider an electric charge as a quantity of fluid which overlies the charged body. With this idea in mind, the surface density at any spot may be considered as the thickness of the fluid at that spot. Independent of such convention, the definition is: The surface density at any point of a charged body is the number of units quantity per unit area at that point.

A convenient instrument for showing the existence and behavior of surface density is shown in Fig. 7. A good sized sheet of tinfoil is mounted on an insulating roller which is supported so that the foil may hang free. Now if a charge of electricity be placed upon the foil it will immediately spread, covering the whole surface. Thus quite a charge may be imparted without giving any great density. If now the tinfoil be wound upon the roller the surface will materially decrease, and since — as will be shown in § 19 — the charge cannot lie within a conductor, the same quantity of electricity must reside on a less surface, hence the density must be greater. That this condition exists can easily be shown by the gold-leaf electroscope, which shows only slight electrification when ap-

proached to the open sheet of foil, but shows much more intense electrification when approached to the foil when rolled up. Or a spark may be drawn from the cylinder of foil which could not be drawn from the sheet.

If a charge be placed upon a conducting sphere and no other charges or conductors be in the vicinity, then the charge is uniformly distributed over the surface. If a

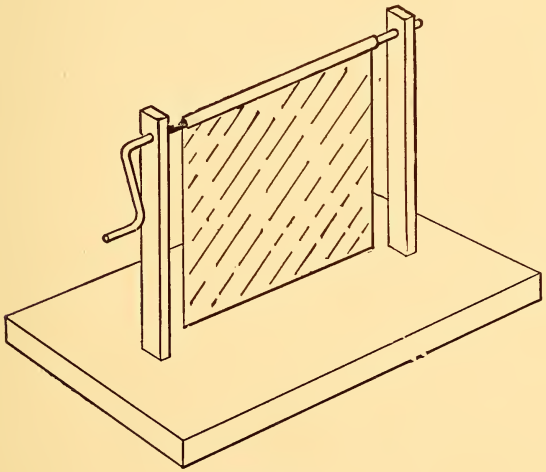

FIG. 7.

sphere have a radius r, its surface is $4\pi r^2$, and if a charge of q units of electricity be imparted to it, then its surface density at any point is $\frac{q}{4\pi r^2}$.

This condition of uniform density may be represented as in Fig. 8 *a*, where the dotted line shows the thickness of the "fluid" on the surface of the sphere. If, however, the body is not spherical then the distribution will not be

uniform, but the surface density will be greatest where the radius of curvature is shortest. For instance, on an ovoid the distribution will be about as in Fig. 8 *b*, where the density is greatest at the sharpest end. Figs. 8 *c* and

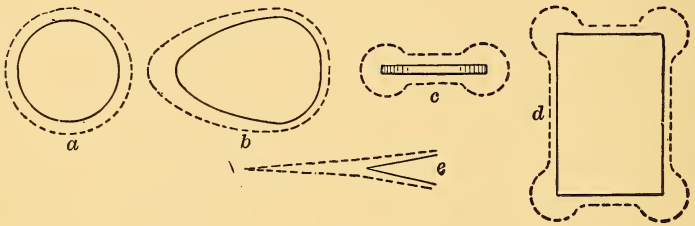

Fig. 8.

d show the distribution on a disc and cylinder respectively. In the case of a point where the radius of curvature is practically 0, the density becomes so great, as in Fig. 8 *e*, that a curious phenomenon occurs, called the *electric wind*. The electrification at the point is so great as to electrify

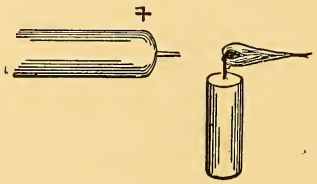

Fig. 9.

the air particles in the vicinity and immediately repel them. The inflowing air particles are in turn charged and repelled, and a stream of air is kept flowing which may be plainly felt by the hand, or may be made to displace a candle flame (Fig. 9). Obviously there is a

constant transfer of charge to the air particles, and the effect cannot be maintained unless the point is kept supplied with electricity, as, for instance, from an electrostatic machine such as is described later. If an electric wind is allowed to blow on an insulating surface, say glass or ebonite, such surface becomes charged with the same kind of electricity as that on the conductor which is causing the wind. The facility with which charges leak from points renders it imperative that electrostatic apparatus be finished in curves and knobs rather than in angles,

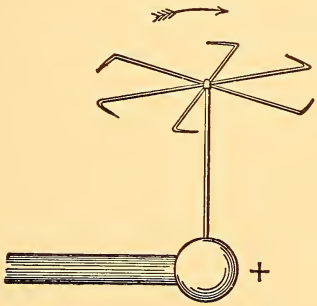

FIG. 10.

edges, or points, save where a discharge is expressly desired. Of course the force of repulsion between the point and the air particles is mutual. This is easily shown by the "electric fly," a few light brass arms radiating horizontally from a hub pivoted on a conducting support, the outer extremeties being pointed and bent horizontally at right angles, as shown in Fig. 10. If now the support be connected by a chain or other convenient conductor to one terminal of an electric machine, the mutual repulsion between the charge on the points and the charges on the air

particles immediately in front of them causes the apparatus to revolve rapidly in the direction opposite that indicated by the points. This "reaction-mill" is an analogue of the Barker's mill known in hydraulic physics.

15. The Electrostatic Field.— When a body is charged with electricity, a strained condition is believed to exist in the ether surrounding the body, and the space in which this strained condition is felt is called the electrostatic field. The field may be represented by *lines of force*

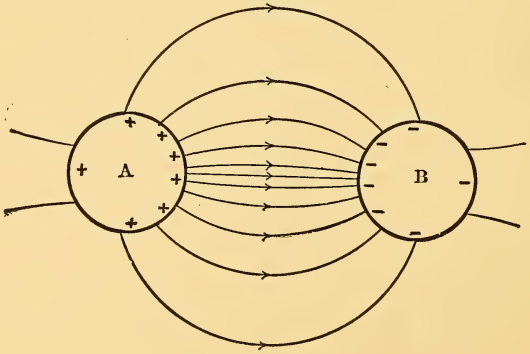

FIG. 11.

emanating from the charged body. A line of force is a line drawn in an electric field such that at any point its direction is the direction of the electric intensity (§ 16) at that point. The field between two spheres is roughly shown in Fig. 11. Such lines may be plotted experimentally by the use of light pivoted straws (Fig. 5), the straws always assuming a position parallel to the lines of force near them. A free positive charge of electricity when placed on a line of force will move along it, and its

direction may be considered the positive direction of that line. A free negative charge placed upon the line will move along it in the opposite direction, and this may be considered the negative direction of the line of force. Since a positive charge is repelled by a positively charged body the positive direction of all lines of force emanating from such a body must be away from it, and for a similar reason the positive direction of lines emanating from a negatively charged body is toward the body. Hence the arrowheads in Fig. 11 point away from the positively charged A toward the negatively charged B.

It may readily be shown that the electrostatic field exists in vacuo and is independent of air or other gas for its existence. This fact is another link in the chain of evidence which unites electrical phenomena with the luminiferous ether.

16. Intensity.—The *intensity* of an electrostatic field at any point is defined as the force in dynes exerted on a free unit positive charge at that point, supposing the condition of the field to be unaltered by the introduction of such free unit charge. Obviously there must be a field of unit intensity at any point one centimeter distant from an isolated unit quantity of electricity (in air), since by definition a unit quantity acts with a force of one dyne on another unit quantity at a distance of one centimeter.

In considering an electrostatic field it is usual to let each line of force represent a force of one dyne per square centimeter. Hence the number of lines per square centimeter of surface measured perpendicular to such lines is a measure of the intensity of the field at that surface.

There can be no intensity at a point within a conductor, for if there were, electricity would be moved until an equilibrium was obtained, which equilibrium would be an indi-

cation that no force tended to transfer electricity, which is to say that the intensity would be reduced to zero.

17. Potential.— To determine the character of a field it is necessary to know not only the intensity at a given point, but also how the intensity varies in the neighborhood of that point. For instance, at two given points the intensity may be just the same, at one, however, due to a small near charge, at the other, due to a large far charge. At these two points the fields would be very different, as is shown in Fig. 12. Let A be one square centimeter perpendicular to the lines of force emanating from

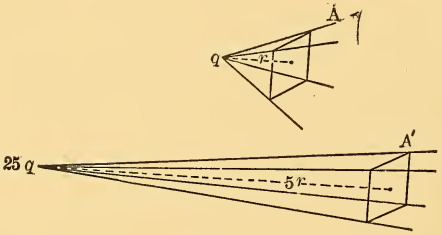

FIG. 12.

a charge q at a distance r. As seen in the figure, let there be one line per square centimeter at A, then at any point in A the intensity is unity. Let also A' be a square centimeter perpendicular to the lines emanating from a charge of 25 q at the distance 5 r. Now since the force between two charges varies directly as the product of the charges and inversely as the square of the distance between them, the force acting on a unit charge at A is exactly equal to the force acting on a unit charge at A'. Therefore the intensity at A' is the same as the intensity at A, and one line per square centimeter must be drawn to A' as shown. It is now readily seen that though the intensities are equal

the fields are quite different, since a slight displacement of A' will be attended with only a slight change in intensity therein, while a slight displacement of A will cause a very marked change of intensity at any point in it.

All these qualities of the electric field are taken into account when the *potential* of a point in the field is considered. The potential at any point in an electric field is the work which must be expended on a unit positive charge to bring it up to that point from an infinite distance. It is evident that if the field were due to a negative charge that the work expended would be a minus quantity, *i. e.,* the unit positive charge would be attracted and would perform work. In this case it is convenient to regard a point in the field as having negative potential. From the foregoing it follows that the *difference of potential* between any two points in an electric field is the work that must be performed on a unit charge to move it from one point to the other.

Obviously a zero potential will exist only at an infinite distance from every charged body. For practical purposes the potential of the earth is taken as zero. This value of zero does not vary because the earth has practically an infinite capacity, hence imparting a charge to it raises its potential only an infinitesimal amount. This assumption of the earth's potential as the zero is analogous to the assumption of the sea-level instead of the center of the earth, as the zero in measuring the heights of mountains.

A surface which is the locus of all points of a given potential is called an *equipotential surface.* A line drawn in one of these surfaces is an *equipotential line.* In moving a charge along an equipotential surface no work is done because the potential does not change. Therefore

there can be no component of the force of the field in the direction of the motion. Therefore all equipotential surfaces must cut every line of force they meet perpendicularly.

When a charge is laid on a conductor and assumes a state of equilibrium, the surface of that conductor is an equipotential surface. For if it were not, then repulsion between the parts of the same charge would cause some of the charge to flow from places of higher (numerically) potential to those of lower potential. But this is con-

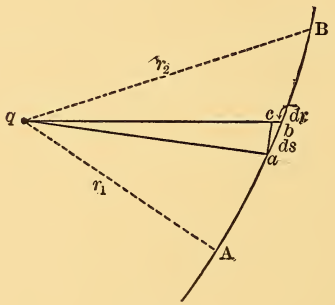

FIG. 13.

tradictory to the first assumption of equilibrium, hence the surface of the conductor is an equipotential surface. This is true even if the surface density of the charge on the conductor varies at different points due to the shape of the conductor or the proximity of other charges. This argument will not hold good however of a body made up of two or more conducting substances, since the mere contact of dissimilar substances causes electrification on the surfaces of contact, positive on the one, negative on the other. Since the surface of a charged homogeneous conductor is equipotential it follows that all the lines of force

emanating therefrom are normal to that surface on leaving it.

Consider an electric charge of quantity q (Fig. 13). At any distance r from it, its force on a unit quantity will be $\frac{q}{r^2}$ dynes. This force resolved into the direction of the path AB, at any point a, is $\frac{q}{r^2}\cos\beta$, where β is the angle, abc. Hence the work done in moving a unit quantity over the infinitesimal distance ds is

$$dw = \frac{q}{r^2}\cos\beta\, ds.$$

But $\cos\beta = \frac{dr}{ds}$

$$\therefore dw = q\,\frac{dr}{r^2}.$$

Integrating over the distance AB,

$$w\ q = \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$w\ q = \left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

By definition the work required to move a unit charge from one point to another is a measure of the difference of potential between those points. Hence the difference of potential between two points distant r_1 and r_2 from a charge q and due to that charge is $q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$. If $r_2 = \infty$ we have, when V represents potential,

$$w\ q = \left(\frac{1}{r_1} - \frac{1}{\infty}\right)$$

$$w = V = \frac{q}{r_1}.$$

This condition complies with the definition of potential, so we see that the potential of any point due to a charge is as the quantity of the charge and inversely as the distance from it. It is well to note here the difference between potential and intensity (§ 16), the condition of the latter at any point being expressed by

$$I = \frac{q}{r^2}.$$

By substituting unit conditions in $V = \frac{q}{r}$ the definition arises:— The unit potential is that due to a unit quantity of electricity at a distance of one centimeter.

As the increase of potential energy of a body moved from a lower point to a higher against the force of gravi-

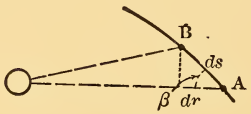

FIG. 14.

tation is independent of the path moved through, so is the difference of electric potential between two points as measured by the work done in moving a unit charge from one point to the other independent of the path the charge is moved through.

In Fig. 14 let V be the potential at a point A and $V + dV$ that at the indefinitely near point B. Let I be the intensity of field at A B and F that part of the force which resolves in the direction A B. The work required to move a unit of electricity from A to B is the (negative) work required to move it from A to an infinite distance plus the work required to move it from an infinite distance,

back to B, that is, $(V + dV) - V$. But the work is also Fds,

$$\therefore Fds = (V + dV) - V$$
$$F = \frac{dV}{ds}.$$

But $F = I \cos \beta$ and $\cos \beta = \frac{dr}{ds}$, therefore $Fds = I\,dr$, so

$$I = \frac{dV}{dr}.$$

18. Flux Through a Closed Surface.— If in an electric field a closed surface be described, if Q be the quantity

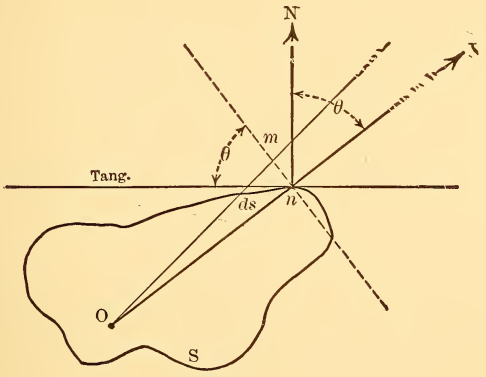

FIG. 15.

of electricity enclosed and if N be the component of the intensity at any element of surface, ds, resolved normally to ds and outward; then the $\iint Nds$ taken over the whole surface is $4\pi Q$. That is, there will emanate $4\pi Q$ lines of force from that surface (§ 16).

To prove this theorem, consider the enclosed quantity Q, Fig. 15, to be concentrated at some point O. Let r

be the distance of the elementary surface ds from O. Let θ be the angle between the direction of the intensity, I, and the normal component thereof, N. Now at ds, $I = \frac{Q}{r^2}$. Therefore $N = \frac{Q}{r^2} \cos\theta$, and $Nds = \frac{Q}{r^2} \cos\theta\, ds$.

Now consider the cone whose apex is at O and whose elements bound the surface ds. If this cone be given the normal base mn at ds, then the area $mn = \cos\theta\, ds$, because one linear dimension of the areas is common, and the other linear dimension is in the ratio of $1 : \cos\theta$. But $\frac{\text{area } mn}{r^2}$ is a measure of the solid angle at the apex of the cone. Hence $\cos\theta\, ds$ is numerically equal to

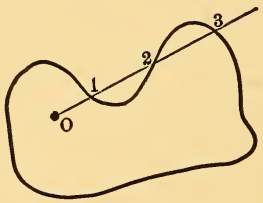

FIG. 16.

the solid angle subtended at θ by ds. This expression is + or − according to the sign of $\cos\theta$. We know that this sign must be + when O faces the inside of the closed surface, and − when it faces the outside.

Now a right line draw from O must cut the surface an odd number of times, as, for instance, in Fig. 16. A cone such as we have been considering which followed the line in Fig. 16 would contribute the term $\frac{Q}{r^2} \cos\theta\, ds$ at the first intersection, $-\frac{Q}{r_1^2} \cos\theta'ds$ at the second, and $\frac{Q}{r_1^2} \cos\theta''ds$ at the third.

But $\frac{\cos\theta' ds}{r_{'}^{2}} = \frac{\cos\theta'' ds}{r_{''}^{2}}$ since they both measure the same solid angle. So two of these terms balance out, and only the first need be considered. Thus, with any degree of re-entrancy, all other terms cut themselves out pair by pair, save the first, which is $Q \times$ the solid angle of the cone.

For the integrated value of the force all over the surface we get $Q \times$ the sum of the solid angles of all the cones filling the space about O, or $4\pi Q$. Therefore,

$$\iint N ds = \iint \frac{Q}{r^2} \cos\theta \, ds = 4\pi Q.$$

which was to be proved.

If the charge Q were at some point O' outside the surface, then the intersections for any cone would be of an even number, contributing an even number of terms alternately + and $-\frac{Q}{r^2}\cos\theta \, ds$, which would reduce the value $\iint N ds$ to 0.

Evidently there must emanate from a unit quantity of electricity 4π lines of force.

19. There is no Charge Within a Conductor.— The relation just derived offers means of reaching conclusions analytically that may be shown to be correct experimentally. Thus we find that there can be no electric charge in a conductor. For, consider an indefinitely small closed surface, described in the body of a charged conductor. There is no intensity at this surface, since if there were the electricity would be moved, while we know that it is in equilibrium. There being no intensity, N must be equal to 0. Hence $\iint N ds = 0 = 4\pi Q$

$$\therefore Q = 0$$

and there is no charge within the body of the conductor.

This fact is shown experimentally by Biot's spheres. A conducting sphere (Fig. 17) is suspended by an insulating thread and a charge imparted to it. Two hollow conducting hemispheres nicely fitting the sphere and mounted with insulating handles are placed so as to almost surround but not touch the sphere. By means of a *proof plane* — a small conducting disc, or sometimes a bead, on an insulating handle, by which a sample of a charge may be taken from a large conductor and examined in the usual way with an electroscope — the charge can be shown to be still present on the sphere. Now close the

FIG. 17.

hemispheres around the sphere, making electrically one sphere of somewhat greater diameter. On removing the hemispheres no trace of charge can be found on the sphere, but it can all be found on the two pieces which for a moment had formed the whole surface of the conductor.

A pair of gold leaves hung inside a wire cage cannot be made to diverge when the cage is electrified, even if in electrical connection therewith. In elaboration of this experiment Faraday constructed a twelve-foot hollow cube of wood, covered it with tinfoil and placed it on insulating supports. He charged it so highly that sparks and brush discharges issued from the corners and edges, and then

placed his most delicate electroscopes inside, but once inside the gold leaves showed no tendency to diverge.

Faraday also constructed a bag of linen gauze — a fairly good conductor to static charges — mounted on an insulated ring, as seen in Fig. 18, with a silk string so fixed to its bottom that it could be readily turned inside out. A charge imparted to the bag was always found on the outside, no matter how frequently or suddenly the inside might be made the outside.

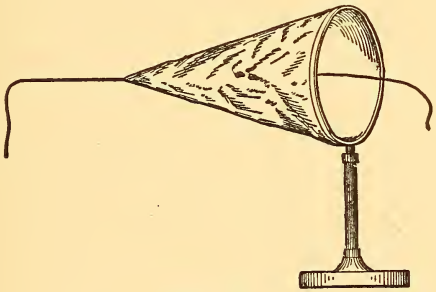

FIG. 18.

20. Charges in a Cavity.— When inside of a conductor there is a cavity, and in this cavity is a charge insulated from the conductor, then on the inside surface of that conductor there will be a charge caused by *influence,* but there is none in the body of the conductor. That the insulated charge and that induced by it on the inside surface of the conductor are exactly equal is shown as follows: Imagine a closed surface described within the body of the conductor and surrounding the cavity. There is no intensity at this surface (§ 16), hence $N = 0$ and $\iint N ds = 0 = 4\pi Q$, hence $Q = 0$. That is, the algebraic sum of the

charges within the surface is 0. Therefore, the insulated charge induces an exactly equal charge of opposite sign on the inner surface of the conductor.

Faraday's experimental proof of this is widely known as his " ice pail " experiment, since he used a metal ice pail

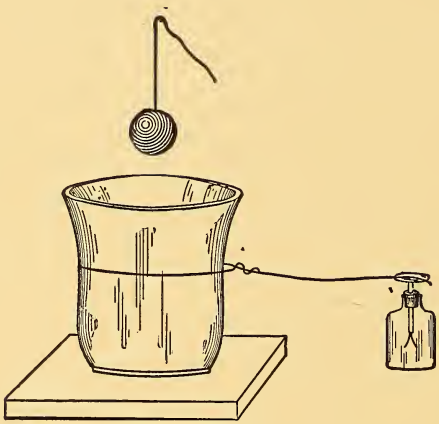

Fig. 19.

in performing it. Let the outside of a metal pail or cylinder, standing on an insulating block, be connected by a wire to an electroscope (Fig. 19). No divergence is observed. Lower a charged ball into the pail by means of a silk thread. The leaves will gradually diverge until the ball is thoroughly within the pail, when they will assume a steady position. Allow the ball to touch the inside of the pail. The leaves will not move. Remove the ball entirely. The leaves remain at their constant divergence, and the ball will be found to have lost all its charge. The explana-

tion is as follows. Suppose the ball to be positively charged. It influences on the inside of the pail a negative charge. This leaves a positive charge on the outside of the pail, which is registered by the electroscope. When the ball touches the pail its charge and the negative charge in the pail neutralize each other. If they were not exactly equal the gold leaves will be diverged or contracted as the positive or the negative charge was in excess.

This experiment shows also that there cannot be a charge influenced in a body without there being an exactly equal and opposite charge in the same body or one electrically connected therewith, which charges flow together and neutralize each other on the removal of the influencing charge, provided that the continuity of the conductor is not disturbed during the experiment, and that no free charges rested upon the conductor before the experiment.

21. Tubes of Force.— Imagine a plane in an electric field normal to the direction of the electric intensity, and a closed figure drawn on such plane. If now a line of force be drawn through each point of this closed figure, such lines will define a tubular surface. Such a tubular surface is called a *tube of force.*

Consider a tube of force of indefinitely small section (Fig. 20). Let a portion of it be made into a closed surface by two normal sections s and s'. Let there be no electricity within this surface. Then $\iint Nds = 0$. Let the intensity be I at s and I' at s'. The whole normal outward force at s is Is and at s' is $-I's'$ (negative because opposite in direction to the lines of force). There can be no force from the other sides of this closed surface, since

there can be no component of intensity normal to a line of force; therefore $Is + (-I's') = 0$, or $Is = I's'$.

By convention we draw as many lines per unit area of s as is the value of I expressed in dynes. But from $Is = I's'$ we see that I' is inversely as s', hence all lines of force passing through s will, when produced, pass through s'. So the intensity of any point in a field is represented by the number of lines of force per normal unit of area at that point.

If equipotential surfaces be drawn, spaced so that there is an infinitesimal difference of potential dV between any

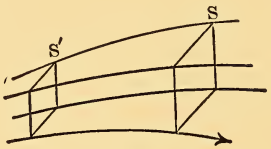

FIG. 20.

two adjacent surfaces, then the number of these surfaces intercepting an infinitesimal length of a line of force, dr, is a measure of the intensity there. For, from section 17, $I = \frac{dV}{dr}$, that is, the intensity at any point is inversely as the distance between the equipotential surfaces, or directly as the number per unit length.

As long as there is no electricity in a tube of force the product Is remains constant. It is evident that there will be a value for I however far the tube may be extended. If $s = \infty$, then I becomes zero, and the tube has an end. Hence we see that a tube and therefore a line of force can begin or end only in a surface charged with electricity.

Consider a tube of force passing from one charged surface to another charged surface of the conductors which form its ends. Prolong the tubes slightly into the conductors and close their ends by planes. The whole tube is now a closed surface. There can be no intensity at those parts of the surface within the conductors, and there can be no component of I normal to the tube outside the conductors, therefore $= Nds = 0$, and the charge on the surface of one conductor embraced by one end of the tube must be exactly equal and of opposite sign to the charge on the surface of the other conductor embraced by the other end of the tube, to fulfil the condition $Q = 0$ in

$$\iint Nds = 0 = 4\pi Q.$$

22. Intensity Outside a Charged Surface.— Let P (Fig. 21) be a point on a charged surface whose density is σ.

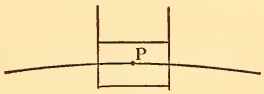

FIG. 21.

Pass a tube of force through the charged surface, cutting out of it a very small surface s. Close the tube inside the conductor by a plane and outside by a surface parallel to the charged surface and indefinitely close thereto. Since the lines of force are normal to s they are also normal to this last surface, hence it is in all respects equal to s. The cylinder thus constructed around P forms a closed surface which encloses the charge σs. If I is the intensity just outside of s, the whole normal force issuing from the closed surface is Is. Therefore $\iint Nds = 4\pi Q$ becomes

$$Is = 4\pi\sigma s,$$
$$\therefore I = 4\pi\sigma.$$

The electric intensity just outside a conductor whose surface density is σ is $4\pi\sigma$.

23. Stress on an Electrified Surface.— In Fig. 22, let P be a point on a conductor whose surface density is σ at which it is desired to find the surface stress. Consider a surface AB about P so small as to be sensibly a plane. On the normal at P consider the points m and n, one inside, the other outside the conductor, and equally distant from P. Now m and n are so indefinitely near together as

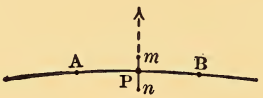

FIG. 22.

to be sensibly one point to all parts of the charge, save that lying on AB. The intensity all over AB, being normal thereto (§ 17), is parallel to Pm. Consider the forces acting at m and n. They may be divided into the force F, due to the portion of the charge on AB, and the force F' due to the rest of the charge. Due to the charge on AB, a unit charge at m would be urged in the + direction with a force F, while at n it would be urged in the — direction with a force F. Due to the rest of the charge, at either m or n, these being now sensibly one point, it would be urged in the + direction with a force F'. Hence the total force, or the intensity, at m is $F' + F$, while that at n is $F' - F$. But we know that the intensity at m is $4\pi\sigma$ (§ 22), while that at n is 0 (§ 19). Hence

$$F' + F = 4\pi\sigma$$
$$F' - F = 0$$
$$\therefore F' = 2\pi\sigma.$$

That is, the rest of the charge exerts a force of $2\pi\sigma$ per unit charge on AB, and in the direction Pm. When σ and s are measured in the absolute units, the total stress on the surface is normal thereto and of a force of $2\pi\sigma \times \sigma s$ dynes. Or the normal outward stress is $2\pi\sigma^2$ dynes per square centimeter.

This stress may be expressed in terms of I just outside the surface, for

$$I = 4\pi\sigma \quad (\S\ 22).$$

Stress per unit area $= 2\pi\sigma^2$

$\therefore$ Stress per unit area $= \frac{1}{8\pi} I^2$.

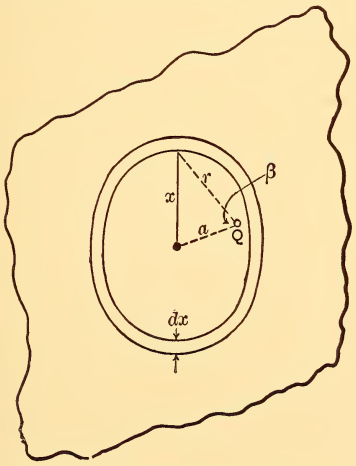

FIG. 23.

The effect of this stress can be qualitatively shown by blowing a soap bubble on a metal pipe. After closing the aperture and holding by means of some insulator, a charge may be imparted to the pipe which is immediately com-

municated to the bubble. The bubble will be seen to expand, the electric repulsion or surface stress overcoming a part of the molecular cohesion.

It is stated that a surface stress of 66,700 dynes per square centimeter is sufficient to cause the electricity to leave the surface, passing through the air in the form of a spark.

24. Attraction Due to an Electrified Plane.— If there be an indefinitely extended plane, a portion of which is represented in Fig. 23, charged to a surface density σ, and if a quantity of electricity Q be at the vertical distance a from that plane, the force of attraction (or repulsion) exerted on that quantity by the plane may be determined as follows:

The attraction between any infinitesimal quantity dq on the plane, for the other quantity Q is (§ 12)

$$dF' = \frac{Q\,dq}{r^2}$$

where r is the distance between the quantities. The component dF of this force which tends to attract Q to the plane is

$$dF = dF' \cos \beta$$

$$= \frac{Q\,dq}{r^2} \cos \beta.$$

But $dq = \sigma\, dS$

and taking a circumferential differential of area,

$$dS = 2\,\pi\, x\, dx$$

$$\therefore dq = 2\,\pi\,\sigma\, x\, dx.$$

Also $$r^2 = a^2 + x^2$$

and $$\cos \beta = \frac{a}{\sqrt{a^2 + x^2}}.$$

Hence

$$dF = \frac{2\pi\sigma a Q x\, dx}{(a^2+x^2)\sqrt{a^2+x^2}}$$

and as the plane is indefinitely extended,

$$F = 2\pi\sigma a Q \int_0^\infty \frac{x\, dx}{(a^2+x^2)^{\frac{3}{2}}}$$
$$= 2\pi\sigma Q.$$

From this it is seen that the attraction exerted on a unit quantity of electricity by an indefinitely extended plane charged to a surface density σ is $2\pi\sigma$ dynes, and is independent of the distance between the unit quantity and the plane.

If a unit quantity of electricity be placed between two indefinitely extended parallel plates equally and oppositely charged, as for instance ordinary condenser plates, the surface density on each will be σ. One plate will attract the unit quantity with a force of $2\pi\sigma$ dynes, the other will repel it with a force of $2\pi\sigma$ dynes. Therefore the resultant force on it will be $4\pi\sigma$ dynes. If V represent the potential of one plate and V' that of the other, then their difference of potential is $V - V'$. But from section 17 their difference of potential is measured by the number of ergs of work necessary to move a unit quantity from one to the other. Work = force × distance, hence

$$V - V' = 4\pi\sigma d$$

where d is the distance between the plates.

CHAPTER III.

CAPACITY.

25. Definition.— The electrostatic capacity of a conductor is measured by the quantity of electricity which must be imparted to it to raise its potential from zero to unity. Thus the capacity of a conductor is not the quantity it can hold without running over, as a quart measure, nor is it the amount it will hold without bursting or discharging or in any way taxing itself to a limit.

The most generally used unit of capacity is the farad. This unit belongs to the electromagnetic system, *i. e.,* a conductor in which one coulomb (ampere-second) will raise the pressure one volt has one farad capacity. Practical capacities are vastly smaller than a farad, so the term microfarad (= .000001 farad) is employed. To reduce a capacity expressed in *c. g. s.* electrostatic units to farads, divide by 9×10^{11}, or what is of more utility, to reduce a capacity in electrostatic units to microfarads, divide by 900,000. This constant is not an exact multiple of 10 because of the difference existing between the electrostatic and electromagnetic systems, which has its origin in the different definitions of a unit quantity. (See § 13.) These relations are discussed more fully in the Appendix.

26. Capacity of a Sphere.— If a sphere of one centimeter radius be charged with a unit quantity of electricity, the potential at any point on the surface will be raised to unity. For the charge may be considered as concentrated at the center of the sphere, and by definition (§ 17),

there is unit potential at a point one centimeter from a unit charge. Hence the sphere has a unit electrostatic capacity. Also if the sphere have a radius r, then it must be charged with r units of electricity to raise its surface potential to unity, and its capacity is equal to its radius expressed in centimeters.

Considering the earth a perfect sphere of 4,000 miles radius, its capacity is

$$\frac{4000 \times 5280 \times 12 \times 2.54}{900000} = 715 \text{ microfarads.}$$

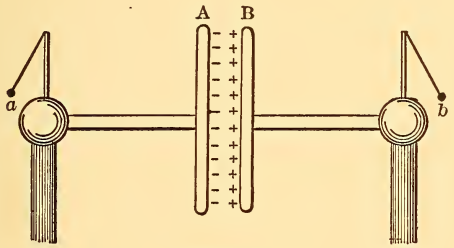

FIG. 24.

27. Principle of the Condenser.— The capacity of any conductor is not dependent solely upon that conductor, but also upon the nature of its surroundings. The relation deduced in the last section is true only for a sphere isolated in air. Consider two conducting discs (Fig. 24) on insulating supports and with rough pith-ball electroscopes, as shown. If the knob a be connected to the negative terminal of an electric machine, the disc A will become negatively charged, and will influence an equal positive charge on the disc B. The negative charge repelled to knob b may then be disposed of by touching b

with a grounded wire, and the connection between the electric machine and *a* may then be severed. We now have equal and opposite charges or quantities on the two conductors. The potentials at all points of a conductor must be the same (§ 17) and the pith-balls will indicate the changes of potential of the conductors by their greater or less repulsion.

If now the discs *A* and *B* be separated, the attraction between the charges will be less, and some of the bound charges will become free, causing an increased indication of the pith-balls; that is, the potential of the conductors has increased. It follows from the definition of capacity that since the potential rises and we know the total quantity has remained constant, the capacity of either of the conductors must have diminished. If *A* were moved to an infinite distance, the potential of *B* would be a maximum for the given quantity and its capacity a minimum, the case of an isolated conductor.

If this experiment be repeated with some dielectric other than air, glass for instance, between *A* and *B*, and more accurate means of measuring the potential be employed, it can be shown that the potential rise for a given charge and a given distance between the conductors is different than it was with air. This difference is further discussed in section 29.

From these facts it seems that the capacity of such an arrangement of conductors and dielectrics depends upon three things: (1) Size and shape of conductors; (2) thickness of dielectric between them; and (3) nature of dielectric.

A useful application of the principle that the potential

of one conductor of a charged condenser increases when the conductors are separated is in the *condensing electroscope* referred to in section 7. In Fig. 6 is shown an ordinary gold leaf electroscope the conductor of which terminates in a metal disc. If now a similar disc, supplied with an insulating handle, and having its surface well covered with varnish, shellac or lacquer, be placed upon the first disc, the two will form a condenser, the varnish being the dielectric. Now a charge of so low a potential as to give no indication with the ordinary electroscope can be examined by imparting some of the charge to the lower disc while the upper disc is grounded by touching with the finger or otherwise. Because of the condenser action of the plates the low potential can impart a relatively large amount of charge to the disc. The source of original charge should first be removed and then the upper disc taken away, thus greatly lowering the capacity of the lower plate, and as the charge on it has no means of escape its potential is greatly increased and gives a satisfactory divergence of the gold leaves.

28. Leyden Jar.—Musschenbrock and Cuneus in the city of Leyden were trying to get a bottle full of electric "fluid." The bottle was half full of water and the "fluid" was to be led in by a long nail extending through the cork into the water. While being charged the bottle was held in the hand. The water then was one conductor, the hand the other and the glass the dielectric between, fulfilling the conditions of the experiment of the last section. On touching the nail with the free hand Cuneus discharged the condenser, getting a "shock."

From this beginning the Leyden jar has developed to its present form, usually a glass jar, Fig. 25, coated on its bottom and about two-thirds of its height, both inside and outside, with tinfoil. A loose fitting wooden cork carries a brass rod terminating at the top in a knob and at the bottom in a bit of brass chain which gives contact with the

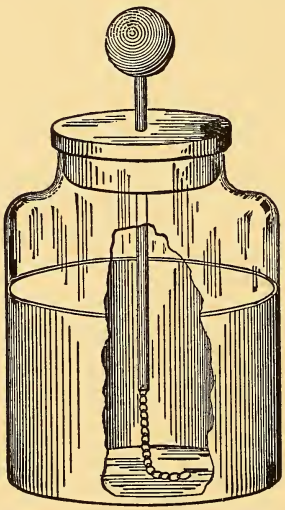

Fig. 25.

inner coating. To charge, the knob is held against the terminal of an electric machine and the outer coating is grounded, either by the hand or a grounded connection. To discharge, a knuckle of the free hand may be presented to the knob giving a spark and a "shock;" or better, the discharging tongs, movable metal legs terminating in knobs and having one or two insulating handles, Fig. 26,

may be laid across the outer coating and the knob. In using the tongs contact should first be made to the coating and then the other leg presented to the knob. If the reverse order is followed the spark will be drawn from the outer coating and may damage it.

A Leyden jar whose diameter is 23 cm. and the coating 33 cm. high has about 2,700 sq. cm. area for each coating. If the glass be .2 to .3 cm. thick, the capacity of the jar will be .004 to .008 microfarads. A condenser consisting of two circular, coaxal, metallic plates, 20 cm. in diameter

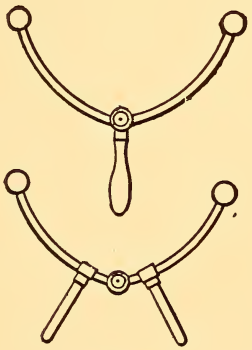

FIG. 26.

and 1 cm. thick, using dry air as the dielectric will have a capacity depending upon the distance between them as follows:

.1	cm.	.000289 microfarad.
.2	cm.	.000149 microfarad.
.5	cm.	.000064 microfarad.
1.0	cm.	.000035 microfarad.

29. Seat of the Charge.— If a Leyden jar be built up of a metal cup, a glass cup and another metal cup, each fitting snugly into the next, the metal cups replace the tinfoil of the ordinary apparatus and may be removed for experiment. If such a jar be charged and then disassembled without discharging, the metal parts show little or no charge. The jar may be reassembled, with other metal cups if desired, and the whole is still found to be charged. This shows that the seat of the charge is on the surface of the glass. The charge on one side being negative and that on the other side positive, they attract each other and hold themselves in place, squeezing the glass between them. This is a real mechanical stress on the glass and may become so severe as to break or pierce it, an accident not uncommon with Leyden jars and which ruins them completely. The utility of the metal coatings would seem to be only to distribute the charge over the glass, or, on discharging, to convey the charge instantly from all over the glass to the point of discharge.

30. Specific Inductive Capacity.— Cavendish first discovered that the capacity of a condenser depended on the dielectric used as well as on its dimensions and arrangement, but Faraday carried the work further and gave quantitative results. If two condensers are exactly similar save that air is the dielectric in one and the substance under consideration is the dielectric of the other, then the ratio of the capacity of the second condenser divided by the capacity of the air condenser is called the *specific inductive capacity* of the dielectric. If the discs of Fig. 24 are immersed in kerosene their capacity will be double that which they have in air. Therefore kerosene has a specific inductive capacity of two (2.03–2.07). The terms

dielectric coefficient and *inductivity* have been employed in this sense. The latter, though little used, seems to be the best because of its brevity and its analogy to *conductivity* as applied to the non-insulators.

While all dielectrics are good insulators, it is not equally true that all insulators are good dielectrics. For instance, paraffin is an excellent insulator, yet its specific inductive capacity is but two, while that of mica is six or eight, and that of glass anywhere from three to ten, according to the observer. There are two reasons for the wide discrepancy in the results of different experimenters on the specific inductive capacity of glass apart from a difference in the physical constitution of the glasses tested. First, the dielectric seems to absorb part of the charge as time goes on, thus increasing the capacity of the condenser. This phenomenon of "elastic fatigue" is treated in the next section. Second, since nothing is an absolute insulator, there is a constant conductive discharge in an electrified condenser. Because of this continuous leak, a method which measures the capacity at the moment of charge will give a much higher "inductivity" for a given dielectric than will a method that requires an appreciable space of time between the charging and the measurement. The specific inductive capacity of nearly perfect vacuum is .999. In determining the specific inductive capacity constant of a dielectric by comparison with an air condenser, the air should be perfectly dry, of a temperature of 0° C. and under a barometric pressure of 760 *m m*.

The following table gives approximately the specific inductive capacities of the more generally used dielectrics:

Air	1
Glass	3 to 7

Ebonite	2.2 to 3
Gutta-percha	2.5
Paraffin	2 to 2.3
Shellac	2.75
Mica	6.6
Beeswax	1.8
Kerosene	2 to 2.7

31. Mechanical Stress Due to Electrostatic Charge.—It is very convenient to consider dielectric stresses as exactly analogous to elastic stresses in other bodies. This hypothesis will explain all the mechanical effects noted in dielectrics, residual charge, electric expansion, and dielectric "elastic limit," each of which will be explained.

The first mechanical effect noticed is the breaking down of the dielectric when a condenser is overcharged. As when a spark is propelled across an air gap, so when a hole is pierced through the glass of a Leyden jar, there must have been real mechanical energy expended in the phenomenon. In such a case the attraction of the charges on the opposite sides of the dielectric strained it so severely that it broke, the same as a drum head, under too heavy a pressure, will rupture.

When a Leyden jar has been charged and discharged, after the lapse of some minutes it may be discharged again, much more feebly but yet noticeably. This phenomenon may be repeated two or three times, each time more feebly and with longer waits between. On charging the jar the molecules of the glass were squeezed together by the attraction between the charges on the opposite sides. Glass is not perfectly elastic, so on discharging the molecules did not at once resume their normal position, but

took some position approximating the normal. After a lapse of time they are prepared to resume more nearly their normal position and a further discharge enables them to do so. Air is nearly perfectly elastic, and an air condenser has no detectable residual charge. A glass fibre when twisted will return almost to normal instantly, and reach its normal only after the lapse of time. A quartz fibre has no such elastic fatigue, but returns all the way to normal at once. A glass condenser has a noticeable residual charge while a condenser which has layers of quartz crystal for its dielectric shows no residual charge.

"Electric expansion" is the term given to the phenomenon exhibited by the Leyden jar of expanding when charged so that its cubic contents is increased. Duter claimed to have shown that this expansion is inversely proportional to the thickness of the glass, and also proportional to the square of the potential difference. If we consider the glass to be subjected to a squeeze along one of its dimensions, *i. e.,* its thickness, it is natural to expect it to expand in its other two directions, that is, in the height and in the circumference of the jar. This was the explanation adapted by Priestly and Volta.

Recent experiments go to show that there is a limit to a dielectric analogous to the elastic limit of metals. If a steel bar be stressed to any point within a certain limit it will on being set free resume its original condition, while if the stress is carried beyond this point the bar takes a permanent set and its elasticity and its ultimate strength are seriously impaired. So also with dielectrics. When subjected to moderate potentials they will return to their original condition, but when the stress is carried beyond the "elastic limit" the insulator takes a permanent "set"

and its future usefulness is impaired. Thus it is quite possible that in testing insulators their value may be seriously reduced by subjecting them to a voltage above their "elastic limit" but below their ultimate break-down point, and the injury not be detected till they are put in service.

32. Dielectric Hysteresis.— When a condenser is charged there is a loss of energy in the dielectric that appears as heat. A small portion of this loss is due to the leakage of charge through the resistance of the dielectric. This passage of electricity is really a current and thus classes the phenomenon as electrodynamic. The energy appearing as heat may be represented as I^2R watts, where I is the current in amperes and R the resistance in ohms. This loss is practically infinitesimal. A much greater loss of energy is due to what seems to be a molecular friction in the body of the insulator, which is quite analogous to the magnetic hysteresis of iron. A physical conception of this action may be had by considering that the dielectric is made up of a number of small particles, adjacent but not touching, which we may call molecules. Each molecule has a small positive charge bound at one end and an equal negative charge at the other. These charges in their turn bind other charges on adjacent molecules. The molecules are irregularly disposed and the extremities carrying the positive charges may point in all directions indiscriminately. Now when a positive charge is laid on one plate of the condenser and a negative charge on the other, these molecules are all swung around by the attraction between unlike charges so that their negative charges will all point toward the positively charged plate. The work thus expended to swing them is not stored either potentially or kinetically but ap-

pears as heat. This work is the loss due to dielectric hysteresis. The student must remember that this reasoning is hypothetical and is introduced as are analogies in other parts of the work, merely to aid in giving a firmer grasp on the subject. There is no independent proof of the existence of such charges on the molecules, nor indeed is there any such proof of the existence of the molecules themselves.

The phenomenon of dielectric hysteresis while of some importance in electrodynamics has but slight influence in electrostatics, but it is treated here as being an essential part of the study of the condenser. The existence of this phenomenon is easily shown. For instance, a paraffin condenser whose ohmic resistance is, say, one megohm will when connected to a source of *E.M.F.* of 100 volts suffer an $I^2 R$ loss in the dielectric thus:

$$I = \frac{E}{R} = \frac{10^2}{10^6} = 10^{-4}$$

$$I^2 R = 10^{-8} \times 10^6 = 10^{-2} \text{ watt.}$$

The temperature rise due to this loss would be so small as to be beyond the range of ordinary measurement. Yet if this source of *E.M.F.* be of reasonably high frequency, 125 ~ (125 complete cycles per second), for instance, the condenser may soon get so hot that the paraffin will melt and run out. If air bubbles had been left in the dielectric in the process of manufacture, the bombarding of the air particles back and forth due to their alternate attraction and repulsion might account for some of the rise of temperature, but this in itself is not of sufficient magnitude to explain the phenomenon.

We have already seen that the best insulators did not

necessarily make the best dielectrics for condensers, because of the difference in their specific inductive capacities, and in dielectric hysteresis we have another factor to consider in selecting a material for such a purpose. Air is a better insulator in the ohmic sense than either oil or mica, but it is inferior to these in specific inductive capacity, in disruptive strength, and in dielectric hysteresis, hence either of the latter is preferable to the former for such use.

33. Calculation of Capacity.— It is almost impossible to calculate directly the capacity of ordinary conductors, but a few of the simpler geometric forms can be treated theoretically.

If C = the capacity of a condenser, V = the difference of potential between the conductors, and Q = the quantity with which it is charged, then by definition of capacity

$$C = \frac{Q}{V}.$$

(a). If we have two concentric conducting spheres (Fig. 27), the smaller of radius r, the larger being hollow and of

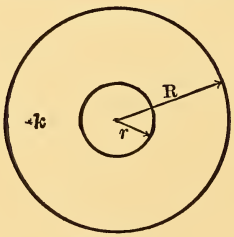

FIG. 27.

internal radius R, and a charge Q rest upon the smaller, then a charge Q will be influenced on the inside of the larger. By section 19 the potential at any point within a closed conductor is the same as that of the conductor, and

by section 17, the potential at the common center of the spheres is to the algebraic sum of each of the neighboring charges divided by its distance or

$$V \text{ at center} = \frac{Q}{r} - \frac{Q}{R}$$

and this is the potential of the inner sphere. Since there is no free charge on the outer sphere its potential is zero and the difference of potential between the conductors is

$$V = \frac{Q}{r} - \frac{Q}{R} = Q\left(\frac{1}{r} - \frac{1}{R}\right).$$

Substituting this in our capacity equation we have

$$C \quad \frac{Q}{V} = \frac{1}{\frac{1}{r} - \frac{1}{R}} = \frac{r\,R}{R - r}.$$

The capacity of two concentric spheres is equal to the product of their radii divided by the difference of their radii.

If instead of air some dielectric whose specific inductive capacity is k be employed, the equation becomes

$$C = k\,\frac{r\,R}{R - r}.$$

(b). To find the capacity of two portions, each of area S and opposite to the other, of two indefinitely extended parallel plates, distant a apart. Let the two plates be charged with equal and opposite quantities of electricity. These quantities will be distributed evenly over the plates. Let the quantity per unit area be q. Being indefinitely extended plates, the influence of their edges is neglected. The potential at any point P on the positively charged plate due to the charge on any circumferential differential of area (Fig. 28) is $\frac{2\pi r\,q\,dr}{r}$, (quantity divided by dis-

tance.) Therefore the total potential at P due to the positively charged plate is the sum of all such differentials of potential over the whole plate or $\int_0^\infty \frac{2\pi r q\, dr}{r}$. Since the plates are oppositely charged, and are a distance apart,

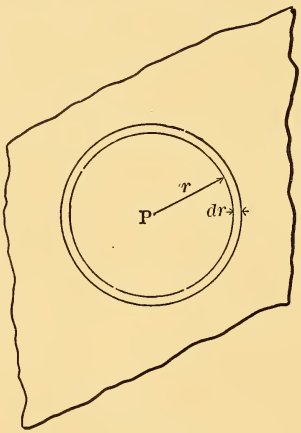

FIG. 28.

the potential at P due to the negatively charged plate is $\int_0^\infty -\frac{2\pi r q\, dr}{\sqrt{r^2+a^2}}$. We may now write the total potential at P.

$$V' = \int_0^\infty \left(\frac{2\pi r q\, dr}{r} - \frac{2\pi r q\, dr}{\sqrt{r^2+a^2}} \right)$$

$$= 2\pi q \int_0^\infty \left(1 - \frac{r}{\sqrt{r^2+a^2}} \right) dr$$

$$= 2\pi q \left[r - \sqrt{r^2+a^2} \right]_0^\infty$$

$$= 2\pi a q.$$

Likewise the potential at a corresponding point on the negatively charged plate is

$$V'' = -2\pi a q.$$

The total difference of potential between the plates is

$$V = V' - V'' = 4\pi a q.$$

The quantity of electricity on the portion S of one of the plates, $Q = Sq$.

Substituting in the capacity equation

$$C = \frac{Q}{V} = \frac{Sq}{4\pi a q} = \frac{S}{4\pi a}.$$

If the plates are of finite dimensions but of great dimensions when measured in terms of the separating distance, the same formula will apply without serious error.

The capacity of two similar plates very near together is equal to the area of one side of one plate divided by 4π *times the distance between them.*

If the separating medium is other than air, the formula becomes

$$C = k\frac{S}{4\pi a},$$

where k is the specific inductive capacity of the dielectric.

(c). To find the capacity of the portions cut off from two indefinitely long coaxal cylinders of radii r and R by two planes perpendicular to the axis and distant l apart. Suppose a charge be distributed over the inner cylinder (Fig. 29) such that there will be a quantity q per unit area. By a reasoning analogous to that employed in the case of the concentric spheres it can be shown that the outer cylinder will be at zero potential and that the potential of the inner cylinder will be the same as that of any point on its axis. The differential of potential at any point

on the axis P is that due to the charge on the circumferential differential of area of the small cylinder divided by its distance, minus the charge on the corresponding circumferential differential of area of the large cylinder divided by its distance, or

$$dV = \frac{2\pi r q\, dz}{\sqrt{r^2+z^2}} - \frac{2\pi r q\, dz}{\sqrt{R^2+z^2}},$$

where z is linear dimension in the axial direction. It may be noted that the quantity on the elementary area of the larger cylinder is expressed in the same terms as that of the smaller cylinder. This is true since the bound charge of the larger cylinder is equal and opposite in sign to that

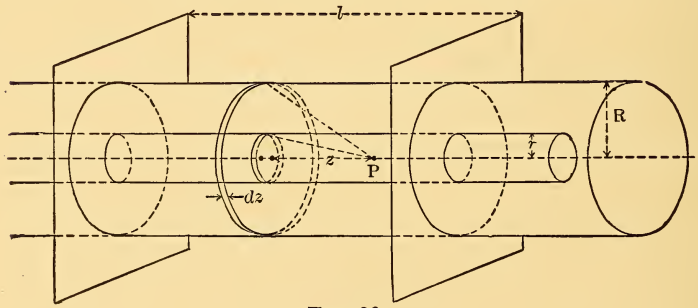

FIG. 29.

of the smaller, and it avoids evolving an expression for the surface density of the larger cylinder. The total potential at P is then (since z is measured in both directions)

$$\begin{aligned} V &= 2\int_0^\infty \left(\frac{2\pi r q\, dz}{\sqrt{r^2+z^2}} - \frac{2\pi r q\, dz}{\sqrt{R^2+z^2}}\right) \\ &= 4\pi r q \left[\log_\varepsilon \frac{z+\sqrt{r^2+z^2}}{z+\sqrt{R^2+z^2}}\right]_0^\infty \\ &= 4\pi r q \log_\varepsilon \frac{R}{r}. \end{aligned}$$

The quantity of electricity on the required portion of the inner cylinder is

$$Q = 2\pi r l q.$$

Substituting, and introducing the dielectric constant

$$C = k\frac{Q}{V} = k\frac{2\pi r l q}{4\pi r q \log_{\varepsilon}\frac{R}{r}} = k\frac{l}{2\log_{\varepsilon}\frac{R}{r}}.$$

A transmission line having considerable capacity requires a certain quantity of electricity to charge it which is not available for use at the far end. In direct current engineering this is of no consequence, since the line once charged remains charged while in operation. In alternating current work the line is charged first positively then negatively, at each reversal of current the line discharging in the direction from which it was last charged. In telephony the frequency of alternation is very high and the capacity of the telephone wires becomes a serious consideration, since a large part of the current that is to actuate a distant instrument is consumed in charging the line while the potential is rising and is discharged when the potential is falling. A standard telephone cable containing two hundred pairs of No. 19 (B. & S.) wires, insulated with dry paper and the whole surrounded by a lead sheath, has a capacity of about .085 microfarad per mile when measured between the two wires of a pair, and about .11 microfarad when measured between one wire and all the others grounded on the sheath. Very recently Prof. Pupin has developed the relations which exist between capacity and inductance, and has shown where and how much inductance is necessary to balance the capacity of telephone lines.

In telegraphy little trouble has been experienced from capacity on ordinary land lines. In submarine work, however, the capacity is enormous, being about one-third microfarad per mile. This multiplied by the transatlantic distance gives a capacity which effectually prevents the transmitting of impulses such as ordinarily are used in telegraphy. Instead of the ordinary open and closed circuit operation the cable is kept charged all the time and impulses are sent by very slightly increasing or decreasing the charge. Such transmission is necessarily slower than the ordinary method.

34. Energy of a Charged Condenser.— The following reasoning can be employed to show the increase of energy in a system of conductors (a condenser for instance) when the charge in such system is increased. Suppose the quantity and the potential to be respectively Q_1 and V_1 at the start, and Q_2 and V_2 at the end of the charging.

We may consider the charging to be accomplished by bringing successive infinitesimal quantities of electricity, dQ, up to the charged conductor from an infinite distance. If the potential of the conductor at this particular time is V, then by the definition of potential, the work done in bringing this infinitesimal quntity up to it is

$$dW = V\,dQ$$

in which both V and Q are variables. At any instant the relation between Q and V can be expressed (from the definition of capacity).

$$Q = CV$$

where C is the capacity of the condenser, hence

$$dW = V\,d\,[CV].$$

Summing up these infinitesimals of work from the original

conditions W_1, Q_1, V_1, to the final conditions W_2, Q_2, V_2,

$$\int_{W_1}^{W_2} dW\, C = \int_{V_1}^{V_2} V\, dV$$

$$W_2 - W_1 = \tfrac{1}{2} C (V_2^2 - V_1^2) = \tfrac{1}{2} C (V_2 - V_1)(V_2 + V_1)$$

But $C V_1 = Q_1$ and $C V_2 = Q_2$, therefore

$$W_2 - W_1 = \tfrac{1}{2} (Q_2 - Q_1)(V_2 + V_1)$$
$$= \tfrac{1}{2} (Q_2 + Q_1)(V_2 - V_1).$$

If the condenser was uncharged at first, then $W_1 = 0$, $Q_1 = 0$, and $V_1 = 0$, and the energy of the system is

$$W = \tfrac{1}{2} V Q = \tfrac{1}{2} C V^2.$$

35. Commercial Condensers.— In practice condensers are made in two classes, one for qualitative work and the other for quantitative experiments. There are four recognized dielectrics employed, air, paraffined paper, glass and mica.

Air condensers are usually made up in some simple geometric form, such as concentric spheres or cylinders, whose dimensions can be readily found and their capacities ascertained by calculation. They are sometimes called *absolute condensers.* Their capacities are small and they are used only in the laboratory.

Standard mica condensers are the best for fine quantitative work. They are very permanent and accurate. In their construction alternate layers of tinfoil and mica are assembled so that every other sheet of tinfoil may be connected to form one conductor while those remaining form the other. This stack is then gently clamped together and left in a heated oven long enough to drive out all moisture. It is then quickly plunged into a hot bath of insulating material, after which it is left to dry in a vacuum chamber.

This process insures the exclusion of all air in the dielectric. Condenser break-downs are frequently due to static electrification of enclosed air, the particles of which, bombarding to and fro with the alternating charges, soften the insulation and lead to its final disruption. If air is in the dielectric, the condenser will get so hot after a few minutes' use on high voltages that the insulating compound will melt and run out. Mica is by far the best dielectric for moderate voltages,— say 250 volts alternating or 1000

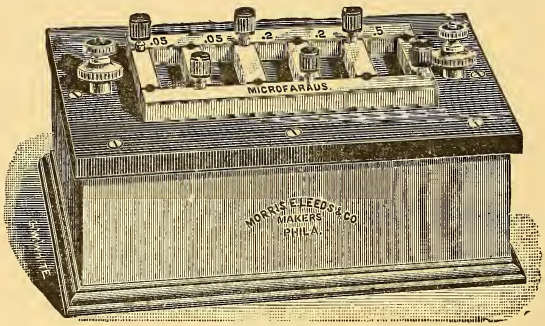

Fig. 30.

volts direct,— extreme accuracy and permanence. It is also the most expensive.

Fig. 30 shows a Leeds standard mica condenser, subdivided into five sections of .05, .05, .02, .02 and .5 microfarads respectively. This instrument is adjusted to an accuracy of $\frac{1}{4}\%$, and is guaranteed to withstand 250 volts alternating indefinitely. The arrangement of the separate condensers is shown in diagram in Fig. 31. This arrangement permits of series as well as of parallel combinations, so that — as will be shown in section 37 — many values from .025 to 1 microfarad can be secured.

Paper condensers are much cheaper and hence can be built for much greater capacities. Save that paper is employed instead of mica their manufacture is very similar to that just described. They may be made to withstand 500 to 1000 volts alternating. In points of insulation, dielectric absorption and dielectric hysteresis they are inferior to the mica condensers. For these reasons they cannot well be used for standards.

The most general use of condensers commercially is in connection with telephone circuits. For this purpose a certain form of paper condenser has become standard. It

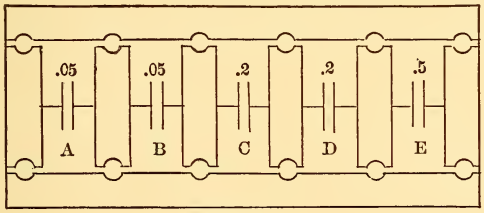

FIG. 31.

consists of two very long strips of tinfoil about eight inches wide and two similar strips of paraffined paper about two inches wider than the tinfoil and somewhat longer. These are alternated and the four rolled up together in a flat form about 10″ by 5″ and of a thickness varying with the capacity, which latter is regulated by the length of the tinfoil strips. This form is treated with a baking process as before described and is then inclosed in a metal case with an insulating top, on which are a pair of terminals respectively connected to the two pieces of tinfoil. Such condensers are made in sizes from .5 to 5 microfarads and are capable of withstanding 500 volts direct.

High potential condensers have thick glass for the dielectric and frequently have copper foil instead of tin foil for the conductors. The thickness of the glass is necessitated by the great dielectric strength required and hence the capacity cannot be great. Such condensers are used experimentally in dealing with high tension, high frequency discharges from induction coils, static machines, and the interrupted spark from the secondary of a high tension step-up transformer.

36. Electrolytic Condensers.—Two rods or plates of carbon immersed in a solution of zinc sulfate will act as a condenser of great capacity if not subjected to a higher potential than about one volt. Two such condensers similar in construction, connected in series, will work satisfactorily at double that pressure and so on. Two lead plates that are unformed, that is, not in the condition of storage battery plates, immersed in dilute sulfuric acid, specific gravity about 1.2, will act as a condenser of great capacity if not charged at more than two volts. These electrolytic condensers have about the same volt-ampere capacity per unit volume as have the electrostatic condensers, but their use is limited because of the extremely low voltages at which they work. Like most electrolytic apparatus these condensers are sloppy and uncleanly, liable to cause damage by upsetting, and prone to unreliability of action. A particularly convenient use to which they can be put is in reducing the sparking at the vibratory devices in induction coils. The action is this: At the moment of breaking the circuit the self-induced E.M.F. which would tend to cause a spark is consumed in charging the condenser which is placed in shunt with the gap. When the circuit is re-established the electricity so stored is given back in useful form.

Another application is in transforming direct continuous current to direct intermittent current to secure greater heating effects. Suppose we have a surgeon's cautery to be heated. Arrange a set-up as in Fig. 32. In series with a lamp between ordinary lighting mains is placed a condenser. In shunt across the condenser is placed a circuit having the cautery and an ordinary vibrator in series. Suppose for a particular case the vibrator is adjusted so as to complete its circuit 1/10 of the time and to leave the

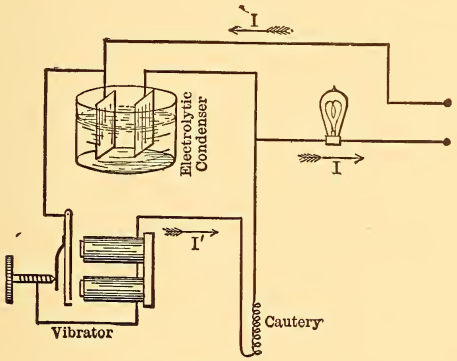

FIG. 32.

circuit open 9/10 of the time. If the resistance R' of the cautery is small in comparison with the resistance R of the lamp,— giving an approximately constant current circuit — and the condenser is of the proper capacity we will have the following condition: The current I will flow through the lamp continuously, but 9/10 of the time it is charging the condenser. Since the whole quantity of electricity must be transferred during the 1/10 part of the time that the shunt circuit is closed and the condenser is discharging,

the current must be ten times as great as that in the lamp circuit, or

$$I' = 10\ I.$$

The heat developed in the cautery during the time of closed circuit is $I'^2 R'$, or averaged through the whole time $\frac{I'^2 R'}{10}$. But

$$\frac{I'^2 R'}{10} = \frac{(10\ I)^2 R'}{10} = 10\ I^2 R.$$

Thus we see that if the cautery were inserted directly in the lamp circuit, the power transformed to heat in the

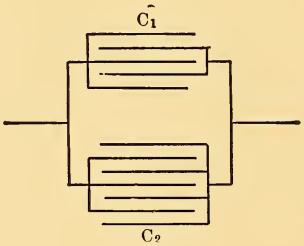

FIG. 33.

cautery would be approximately only 1/10 as much as with the circuit shown.

37. Connection of Condensers.— If two condensers are connected in parallel as in Fig. 33, it is evident that the combination is just equal to one larger condenser whose capacity is the sum of the single capacities.

If two condensers of capacities C_1 and C_2 are connected in parallel, their joint capacity $C = C_1 \times C_2$.

If two condensers of capacities C_1 and C_2 are connected in series as in Fig. 34, a little consideration will show that

when charged they must hold equal quantities of electricity, since any charge influenced on the right member of the left condenser must be balanced by an opposite and *equal* charge on the left member of the right condenser. But as they are equally charged, their differences of potential will be, by definition, inversely as their capacities. If Q be the

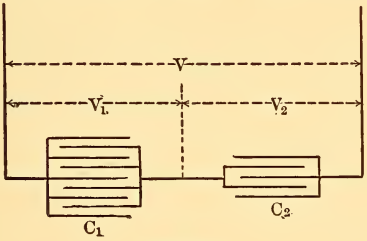

FIG. 34.

quantity of charge, and V the potential at which it is impressed, and C their joint capacity, we have:

$$Q = V_1 C_1 = V_2 C_2 = V C \ldots\ldots\ldots (a)$$

also $$V = V_1 + V_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

Solving (a)

$$V_1 = \frac{V C}{C_1}$$

and

$$V_2 = \frac{V C}{C_2}$$

Substituting in (b)

$$V = \frac{V C}{C_1} + \frac{V C}{C_2}$$

or

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

If two condensers are connected in series, their joint capacity is equal to the reciprocal of the sum of the reciprocals of their single capacities.

The capacity of a series of condensers is always less than the capacity of any one of them taken singly.

It is interesting to note that the formula for capacities in series is analogous to the formula for resistances in parallel, while that for capacities in parallel is analogous to that for resistances in series.

The capacity of a condenser of ordinary design can be approximately calculated by the following formula:—

$$C = .000225 \frac{An}{t} k$$

where

C = the capacity in microfarads,

A = the area of dielectric between two conducting plates in square inches.

n = the number of sheets of dielectric.

t = the thickness of the dielectric in mils (1 mil = .001 inch),

and k = the specific inductive capacity of the dielectric. The values of k for some of the more generally used dielectrics are given on pages 53 and 54.

CHAPTER IV.

EXPERIMENTAL MEASUREMENT OF CAPACITY.

38. Measurement of Capacities.— Whether the dielectric of a condenser is solid or fluid, the capacity is modified by conduction and by dielectric hysteresis, and is some function of the time of charging and the time between charging and measurement. Where these conditions are of any magnitude the capacity can be closely determined only by some method making use of rapid periodic charging and discharging. The direct conductivity of a condenser may be determined by the galvanometer as in ordinary insulation tests. Furthermore the capacity of the wires used in connecting up the condenser may be of such magnitude as compared to the capacity to be measured as to render it necessary to determine their capacity separately and subtract it from the observed capacity to give the true capacity of the condenser.

In the electromagnetic system a capacity has the same dimension as the quotient of a time divided by a resistance, hence the suitable measurement of a time and a resistance will determine a capacity. If the time be measured in seconds and the resistance in *c. g. s.* units the capacity will be expressed in *c. g. s.* units, while if the time and resistance be measured in seconds and ohms respectively, the capacity is given in farads.

An application of this principle is shown in Fig. 35, where x and y are the coils of a differential galvanometer, and T a tuning-fork contact maker charging and discharging the condenser k at a known frequency. The condenser

charges at the battery pressure, and discharges through the coil y. Coil x is constantly excited by a shunt current. Now, if the resistances ab and bc are so chosen that the galvanometer does not deflect, and if we let

R_1 = resistance of ab
R_2 = resistance of ac
R_g = resistance of coil x

and n = number of vibrations per second of the tuning fork, then the capacity of the condenser K is

$$C = \frac{R_1}{n R_2 (R_1 + R_g)}.$$

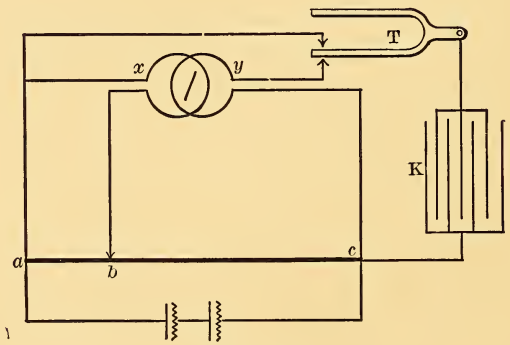

FIG. 35.

The Wheatstone's bridge may be adapted to the measurement of capacity. In Fig. 36 one arm of a bridge is broken to make alternate contacts with a tuning-fork contact maker, which has the condenser to be measured in series with it, the arrangement being such that at one extremity of the fork's vibration the condenser is charged while forming one arm of the bridge while at the other extremity this arm is opened and the condenser is dis-

charged through a short circuit. Letting a, b, c, g, and r represent respectively the resistances of their branches, and n the number of vibrations per second of the tuning fork, then the capacity of the condenser is

$$C = \frac{a}{n\,b\,c} \cdot \frac{1 - \dfrac{a^2}{(a + c + g)(a + b + r)}}{\left(1 + \dfrac{a\,r}{c\,(a + b + r)}\right)\left(1 + \dfrac{a\,g}{b\,(a + c + g)}\right)}.$$

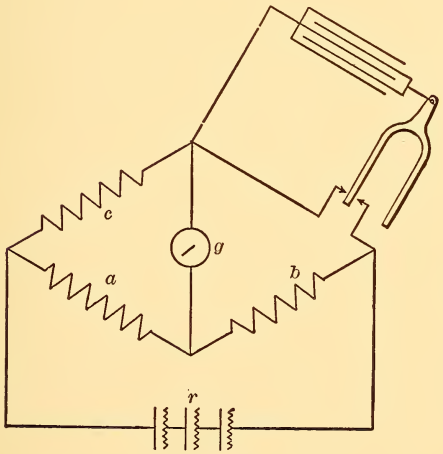

FIG. 36.

If b, c, and g are about equal, the formula reduces approximately to

$$C = \frac{a}{n\,b\,c} \cdot \frac{1}{1 + \dfrac{ag}{b\,(c + g)}}.$$

Further, if b and c are taken very large (10^5 to 10^7 ohms), a and r relatively very small (10 to 10^3 ohms), and

g intermediate between them, (say 10^4 ohms), then approximately

$$C = \frac{a}{n\,b\,c}.$$

As in measuring resistances in the Wheatstone's bridge, the battery connections should be reversed between dif-

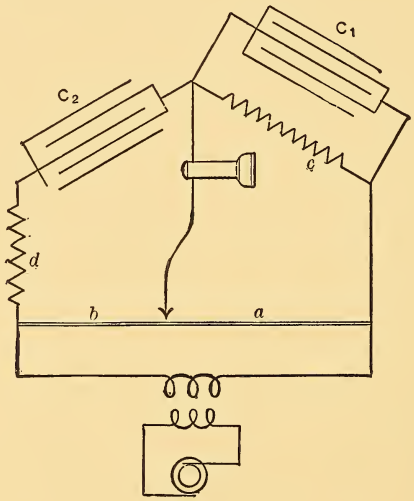

FIG. 37.

ferent observations to avoid error due to thermal E. M. F. generated in the bridge arms.

Another bridge method which determines simultaneously two capacities is shown in Fig. 37. Shunted around arm c is a condenser C_1, while in series d is a condenser C_2. Arms a and b are in a bridge wire on which plays a sliding contact going to what is usually the galvanometer circuit. The "optical telephone" is however used, since it is to detect alternating currents. This is a telephone upon

whose diaphragm is affixed a small mirror which reflects an illuminated slit upon a screen. The image will, of course, move violently for very small displacements of the diaphragm. An alternating E. M. F. is applied to the terminals of the wire $a\ b$, and the resistance c and the sliding contact are manipulated until no current flows in the telephone. If a, b, c, and d represent respectively the resistances of their branches, and f equals the frequency of the alternating E. M. F. (found by multiplying the number of revolutions per second of the generator by half the number of its poles), then

$$C_1 = \frac{1}{f}\sqrt{\frac{c\,b - a\,d}{c^2\,a\,d}},$$

$$\text{and } C_2 = \frac{1}{f}\sqrt{\frac{a}{d\,(b\,c - a\,d)}}.$$

39. Comparison of Capacities.—In determining a capacity it is usual to compare it with a known capacity, this being as a rule quicker and more simple than measuring its absolute capacity.

The simplest application of this principle requires merely a cell of constant E. M. F. and a ballistic galvanometer whose constants need not be known, a known capacity C_1, and the capacity to be compared C_x. The procedure is as follows: Charge C_1 from the source of constant E. M. F. and discharge through the galvanometer, noting the deflection θ_1. When the galvanometer is again at rest, repeat with C_x, noting the deflection θ_x. Then

$$\frac{C_1}{C_x} = \frac{\sin \frac{1}{2}\theta_1}{\sin \frac{1}{2}\theta_x}.$$

This method is quick and fairly accurate if a good condenser is used as C_1. The most accurate results are ob-

tained when the condensers are of almost the same capacity.

Fig. 38 shows the method of Cohn and Arons. A quadrant electrometer connected "in quadrant" is used, *i. e.*, the very small pressure to be observed is used to charge one diagonal pair of quadrants, while the other pair is kept at zero potential by an earth connection. The needle is charged to a high potential either by a many cell battery or a highly charged Leyden jar, the remaining side of the battery or jar being connected to earth. This is the hetrostatic method of operation described in section 43. This instrument must be calibrated so that the potential at which

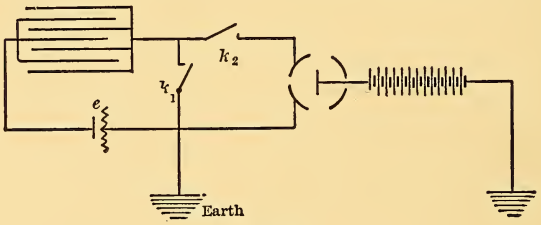

FIG. 38.

the quadrants are charged is known for any given deflection. In the set-up a cell of known E. M. F. has one pole connected to the earthed pair of quadrants and the other pole to the condenser; the remaining pair of quadrants is connected to the other side of the condenser through a key k_2. A third branch containing a key k_1 shunts the electrometer and k_2. The two keys are designed to be opened in quick but measurable succession, and this is accomplished by a pendulum. Now if

E = E. M. F. of the cell in volts,

V = potential of quadrants after opening the keys,

t = time in seconds between opening k_1 and k_2,
R_c = ohmic resistance of the condenser,
c = capacity of one pair of quadrants in farads,
and C = capacity of the condenser, in farads, then

$$(C + c) R_c = \frac{t}{\log \frac{E}{E - V}}.$$

If c and R_c are known this is an absolute measure of C, but if they are not, the experiment is repeated twice, first with a condenser of infinite ohmic resistance (an air condenser whose capacity C' can be determined by linear measurement) placed in parallel with C, and second with a large known resistance R placed in parallel with the original condenser. From the second and third observations we have

$$(C + C' + c) R_c = \frac{t}{\log \frac{E}{E - V}}$$

and

$$(C + c)(R_c + R) = \frac{t}{\log \frac{E}{E - V}}.$$

From these three equations values for $\frac{C}{C'}$, $\frac{c}{C'}$, and R_c may be obtained. This method is free from errors due to conductivity and dielectric hysteresis in the condenser.

De Sauty's bridge method is shown in Fig. 39. Two condensers occupy two arms of the bridge and two resistances — which must be high to attain accuracy — are in the other two. Any delicate galvanometer may be used. A switch is arranged so that c may be connected to b, thus charging the condensers, or to a thus discharging them. When the resistances are so adjusted that no deflection

occurs at charge and discharge then

$$C_1 : C_2 :: R_2 : R_1$$

This method is independent of the battery voltage, in fact a source of alternating E. M. F. may be substituted for the battery and switch if the galvanometer is replaced by a telephone, an electrometer or an electrodynamometer.

Thomson's method is somewhat better. As shown in Fig. 40 the battery circuit is uninterrupted and a key is opened and closed in the galvanometer circuit. When this causes no movement of the needle, then

$$C_1 : C_2 :: R_2 : R_1$$

If one of the condensers, C_1, has a material conductivity

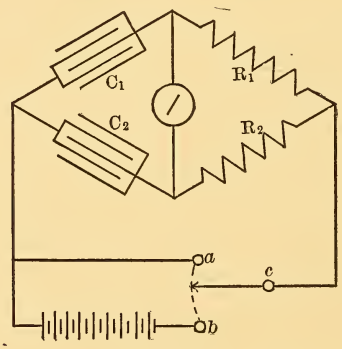

FIG. 39.

its capacity can still be measured by using the optical telephone in place of the galvanometer and key and applying a sinusoidal E. M. F. instead of a battery E. M. F. A resistance R_3 must also be placed in series with the other condenser C_2. Then if R_c is the ohmic resistance of the

condenser C and f is the frequency of the impressed E. M. F. then

$$R_c = \frac{1}{f^2 R_3 C_1 C_2}$$

and the ratio of the capacities

$$\frac{C_1}{C_2} = \frac{R_2}{R_1} - \frac{R_3}{R_c} = \frac{R_2}{R_1} - f^2 R_3^2 C_1 C_2.$$

The last member of the right side is a correction member for which an approximate value for $C_1 C_2$ is sufficient.

This method as well as others using an alternating charge and discharge of high frequency is apt to give

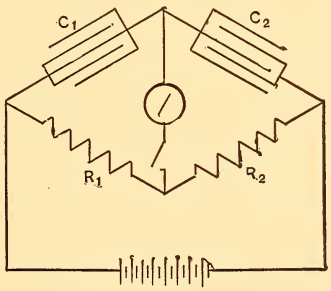

FIG. 40.

erroneous results when applied to condensers which have a considerable charging period, that is, require considerable time to receive their full charge after the E. M. F. has been impressed. Cables, particularly submarine, have a very considerable charging period.

To determine if two condensers are of the same capacity by means of alternating currents and the telephone, the comparison resistances of the de Sauty method may be made infinite, reducing the set-up to the simplicity of Fig.

41. If the capacities are equal the diaphragm of the telephone will not move. This method is very rapid and quite accurate for testing a number of capacities of supposedly the same value. One of the capacities may be made variable and calibrated and the others all balanced up against this. Another arrangement is shown in Fig. 42.

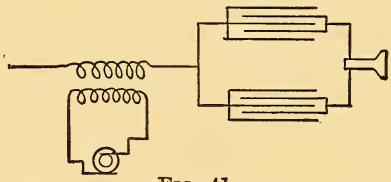

FIG. 41.

The comparison of capacities by separation of the charge was the method used by Farady in his original investigations. Let Q represent the quantity of electricity held by a condenser of C_1 capacity when charged to a potential E. Now disconnect the condenser from the source of E. M. F.

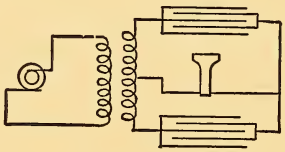

FIG. 42.

and connect it to a second condenser which was previously unchanged. If q represent the quantity now held by the first condenser, and C_2 is the capacity of the second, then

$$\frac{C_1}{C_2} = \frac{q}{Q - q}.$$

If C_2 is very small as compared with C_1, C_1 may be dis-

charged into C_2 n times and C_2 entirely discharged after each of its n charges. Then if q is the quantity in C_2, after the nth discharge,

$$\frac{C_1}{C_2} = \frac{\sqrt[n]{q}}{\sqrt[n]{Q} - \sqrt[n]{q}}.$$

This method may be used to compare the capacities of cables with normal (air) condensers, but is liable to great errors due to leakage unless the successive discharges are made very quickly.

The capacity of any condenser can be directly measured with a good degree of accuracy by means of a suitable calibrated ballistic galvanometer as described at the end of section 41.

CHAPTER V.

INSTRUMENTS USED IN ELECTROSTATCS.

40. Ballistic Galvanometer.—An instrument for detecting and measuring electric currents or quantities by virtue of the electromagnetic reactions of such currents is called a *galvanometer*. Of these there are two general classes, those in which the current in a fixed coil deflects a pivoted magnet,— as for instance, a compass needle,— and those in which the current in a coil suspended in a fixed magnetic field causes the said coil to move. An instrument of this nature having no permanent magnet, but consisting of a fixed coil and a suspended coil, is generally called an *electrodynamometer*. Electrostatics play no part in the operation of this instrument, but it is extensively used in the observation of electrostatic phenomena.

A galvanometer whose moment of inertia is so great that the entire transitory current due to passing a charge through it may be considered to have passed before the needle has moved sensibly from its position of equilibrium is called a *ballistic galvanometer*. When such an instrument is used, not the steady deflection is read, but the extent of the first *throw* or *kick*, and as the motion is slow, the extent of such throw is easily observed. In the early types of galvanometers where the earth's magnetism was the controlling force, a needle was made ballistic by weighting it, or by making it in the form of a sphere. In the modern D'Arsonval or swinging coil type, slow motion is secured by making the coil sufficiently heavy.

Like the theoretical pendulum, an unimpeded galvanom-

eter needle or coil would oscillate forever, once it had been started. Practically there are several causes which tend to bring the needle to rest, among which are; — the friction of the air, the friction of the pivot or the only partial elasticity of the suspending fibre as the case may be, and most of all, the generation of electric currents in adjacent conductors due to the changing magnetic field. This last influence is most clearly shown in the case of a suspended coil galvanometer. If this coil be moving in the permanent magnetic field of the instrument, it will act like the armature of a dynamo, and have electromotive force set up in it, due to its conductors cutting magnetic lines of force. If the coil be on open circuit, no current can flow, and the retarding or *damping* effect will be small. If however the coil be connected with other apparatus so that it forms part of an electric circuit, then current will flow due to this electromotive force, such current doing work in heating the wires and absorbing energy from the moving coil till the latter is brought to rest. The extent of the damping will depend on the resistance of the circuit; the less the resistance, the greater the current and the heavier the damping. Galvanometers of the swinging magnet type can be similarly damped by bringing masses of copper in the vicinity of the moving needle in which Foucault or eddy currents may be generated, thus wasting the energy. Galvanometers for less delicate work may be damped by having a projection from the moving member equipped with fan-like vanes or dipping into mercury or some more or less viscous oil. The method of damping by supplying a suitable outside circuit is the most usual with modern instruments. To bring the coil to rest preparatory to taking a reading, it is customary to short-circuit the wires

leading to the galvanometer, thus giving the heaviest possible damping.

41. Calibration of Ballistic Galvanometer.— To calibrate a ballistic galvanometer is to determine an expression involving Q, the quantity of electricity passed in a transitory current through the galvanometer, and β, the throw of the needle due to this quantity, such that the quantity passed can be determined for any observed throw.

As shown in Gray's "Electricity and Magnetism" or in Maxwell's "Electricity and Magnetism," from a consideration of the equations of mechanics relative to the motion in bodies due to various forces, the equation for a damped galvanometer of the magnetized needle type can be written

$$Q = \frac{H}{G} \frac{T}{\sqrt{\pi^2 + \lambda^2}} \sin \frac{\beta \varepsilon^{\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}}}{2}$$

in which

H = the horizontal intensity of the magnetic field in which the needle swings,

G = the moment of the forces with which a unit current in the coils acts upon the needle when both coils and needle are in the same vertical plane,

T = the time of one *complete* oscillation, as for instance, from one maximum elongation to the next maximum elongation in the same direction.

λ = the logarithmic decrement. The amplitudes of the needle's oscillations are in a diminishing geometrical progression. λ is the natural logarithm of the common ratio of this progression. Thus if one amplitude (to the right) be a_1 and the next (to the left) be a_2 then $\lambda = \log_\varepsilon \frac{a_1}{a_2}$.

The only arbitrary assumption included in the above formula is that the damping is proportional to the velocity of the swing of the needle.

If the deflection of the needle is small, then the sine of the angle can be taken as equal to the angle, and the above formula becomes,

$$Q = \frac{H}{G}\frac{T}{2\sqrt{\pi^2+\lambda^2}}\,\beta\varepsilon^{\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}}.$$

If the logarithmic decrement is small then $\tan^{-1}\frac{\pi}{\lambda} = \tan^{-1}\infty = \frac{\pi}{2}$ nearly

and

$$\varepsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}} = \varepsilon^{\frac{\lambda}{2}} = 1 + \frac{\lambda}{2} + \frac{\lambda^2}{4} + \frac{\lambda^3}{48} + \ldots.$$

but as λ is small the higher powers may be omitted and the equation last written reduces to

$$Q = \frac{H}{G}\frac{T}{2\pi}\left(1+\frac{\lambda}{2}\right)\beta.$$

No sensible error is introduced by using this last equation if $\lambda < \frac{1}{2}$ and $\beta < 30°$ or $\frac{\pi}{6}$.

If there is no damping at all, $\lambda = 0$ and the original equation reduces to

$$Q = \frac{H}{G}\frac{T}{\pi}\sin\frac{\beta}{2} = \frac{H}{G}\frac{T}{2\pi}\beta.$$

if the deflection is small.

To calibrate a ballistic galvanometer by means of the above equations it is necessary to know $\frac{H}{G}$, T and λ. The

value of $\frac{H}{G}$ may be determined by passing a uniform current through the galvanometer and observing the steady deflection. Assuming, as is the case with most galvanometers, that the current is proportional to the tangent of the angle of deflection

$$i = \frac{H}{G} \tan \delta$$

where δ is the deflection and i the current in *c. g. s.* units. If the current be measured in amperes, I, then

$$\frac{H}{G} = \frac{10\,I}{\tan \delta}.$$

The current I will be very small and can best be measured by the fall of potential across some large and known resistance in the circuit, perhaps the resistance of the galvanometer itself, but better if possible, some much larger resistance in series therewith. The fall of potential must be measured by some delicate means, such as balancing against a Clark standard cell on a potentiometer.

T, the period of oscillation, can be determined with any degree of accuracy by taking the time of a sufficient number of oscillations and dividing this time by the number of oscillations. These are complete vibrations, that is, from one extremity of the swing to the other and back again to the first.

The logarithmic decrement, λ, is not a constant in any one galvanometer, but is some function of the circuit in which the galvanometer is included. For, as explained in a previous paragraph, the amount of damping depends upon resistance of the circuit, being approximately inversely thereas. As suggested above, two consecutive amplitudes could be observed and the natural logarithm of

the ratio of the first to the second would give the value of λ. More accuracy however can be obtained by reading a number of successive elongations. These will form the terms of a decreasing geometrical series, and if a be the mth term, and a_n be the nth term the natural logarithm of the common ratio of the series is

$$\log_\varepsilon \textit{ratio} = \lambda = \frac{1}{n-m}(\log_\varepsilon a_m - \log_\varepsilon a_n).$$

By letting the different observed values be in turn a_m and a_n, from a comparatively few observations a number of values of λ may be determined, whose average may be assumed as the correct value of λ. This process should be repeated with different resistances in the circuit, and a curve plotted with values of R and λ respectively as ordinates and abscissæ. Then in any experiment, with a knowledge of the resistance in circuit the value of λ can be determined without further calculation.

It must be remembered that in all the foregoing by "deflection" is meant the angular displacement of the needle from its position of equilibrium. If a mirror galvanometer, throwing a spot of light on a straight scale is used, then the scale reading is proportional to the tangent of twice the angle of deflection.

A galvanometer may also be calibrated by the *inductor method,* which is as follows: If a coil of wire of n turns be rotated in a magnetic field so as to cut lines of force, an E. M. F. will be induced therein whose value in volts (see Sheldon's Dynamo Electric Machinery, § 13), is

$$E = \frac{n}{10^8}\frac{d\varphi}{dt}$$

where $\frac{d\varphi}{dt}$ is the rate of cutting of lines of force. If such

a coil be part of a circuit including the galvanometer with a total resistance of R ohms, then the current

$$I = \frac{E}{R} = \frac{n\,d\varphi}{10^8\,R\,dt}$$

from which

$$I\,dt = \frac{n\,d\varphi}{10^8\,R}$$

Integrating between the limits $\int_0^t dt$ and $\int_0^{\varphi} d\varphi$,

$$I\,t = Q = \frac{n\,\varphi}{10^8\,R} \text{ coulombs.}$$

φ is the number of lines of force cut by the coil which are active in producing E. M. F. If any line of force is cut once by a part of the coil, and is then cut a second time by the same part (not the opposite side), the E. M. F. of the second cut opposes that of the first, and such a line must not be counted as active in producing E. M. F. If the inductor be a coil of n turns inclosing an area of A square centimeters, if it be placed perpendicular to the direction of the horizontal component H of the earth's magnetic field, and if on being revolved 180° in that field it produces a kick a in the galvanometer then

$$\varphi = HA$$

and

$$Q = \frac{2\,n\,HA}{10^8\,R}.$$

(As each turn cuts each line twice, once by its upper half, and once by its lower half, the number of conductors cutting lines is twice the number of turns.) But from the previously written approximate equation of the galvanometer

$$Q = \frac{H}{G}\frac{T}{2\pi}\left(1 + \frac{\lambda}{2}\right)a.$$

Then if K is the constant of the galvanometer

$$K = \frac{H\,T}{2\,\pi\,G} = \frac{2\,n\,H\,A}{10^8\,R\,(1+\frac{\lambda}{2})\,a}.$$

The formula for this galvanometer can now be written simply

$$Q = K(1+\tfrac{\lambda}{2})\,\beta$$

and if the instrument is not damped,

$$Q = K\beta.$$

The method just outlined is called the "earth inductor" method. The chief objection to it is that the value of H is difficult to determine, besides which it is not a constant as to time or place. The "current inductor" method avoids this difficulty. Let a long solenoid be formed of closely wound wire, there being n_1 turns per centimeter length of solenoid. If this is placed east and west and a current of I amperes be passed through it, the number of magnetic lines of force per square centimeter cross section set up therein is

$$\frac{4\,\pi\,n_1\,I}{10}$$

and if the mean area of the solenoid be A, the total flux

$$\varphi = \frac{4\,\pi\,n_1\,I\,A}{10}.$$

If a small test coil of n_2 turns be wound closely around the solenoid and with the galvanometer forms a circuit of R ohms total resistance, then the quantity of electricity passed through this circuit on starting or stopping the current I_1 in the solenoid is (see Sheldon's Dynamo Electric Machinery, §§ 13, 16.)

$$Q = \frac{n_2\,\varphi}{10^8 R} = \frac{4\,\pi\,n_1\,n_2\,I_1\,A}{10^9 R}$$

from which, as before, the constant of the galvanometer,

$$K = \frac{H\,T}{2\,\pi\,G} = \frac{4\,\pi\,n_1\,n_2\,I_1\,A}{10^9\,R\,(1 + \frac{\lambda}{2})\,a}$$

where a is the kick corresponding to the current I_1 in the solenoid.

This method is especially convenient for calibrating D'Arsonval galvanometers, which do not necessarily follow the ordinary ballistic formula. Here various currents may be passed through the solenoid and the quantities corresponding to the various kicks determined. A curve can then be plotted between the two which can be used subsequently

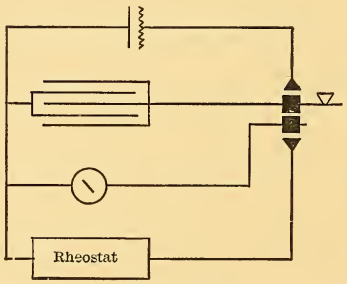

FIG. 43.

for all observations where the external circuit has the same resistance as when the calibration was made. For other resistances other curves must be drawn.

Still another method of calibrating a ballistic galvanometer is by means of the charge of a standard condenser.

To accomplish this an arrangement of apparatus such as is shown in Fig. 43 is employed, the particular feature of which is the quadruple contact key. This key is normally

held up against a contact. In this position the galvanometer circuit is open and the condenser is in series with a charging battery of constant potential. As the key is pressed down three things occur: First, the battery circuit is broken; second, the condenser is discharged through the galvanometer; and third, the galvanometer circuit is closed through an appropriate amount of resistance in a rheostat to form the damping circuit.

Now the potential, E, to which the condenser was charged may be determined; the condenser capacity, C, is known; and the kick, a, is observed.

$$\text{Then} \quad Q = E\,C = K\,a$$

$$\therefore K = \frac{E\,C}{a}$$

where K is a constant directly reducing galvanometer deflections to quantity units according to the formula $Q = K\beta$. This assumes what is practically true of modern D'Arsonval galvanometers, that the throws are proportional to the quantities causing them where the resistance of the damping circuit is the same in each case.

42. Kelvin's Absolute Electrometer.—Two discs oppositely charged will attract each other according to a definite law, and hence can be made to measure the difference of potential between them. The attractive force exerted by a charged point varies inversely as the square of the distance, because the lines of force emanate radially, while from an indefinitely extended plane the lines are parallel and the force is independent of the distance (§ 16). In Kelvin's *attracted-disc electrometer* there are two discs and a ring, arranged as in Fig. 44. B is a balanced disc closely surrounded by the fixed guard-ring G. These two are charged to the same potential and are practically one plate. The

lines of force may be considered to emanate perpendicularly from all parts of this plate save those parts near the edge, where the lines become divergent. Hence it may be assumed that the lines emanating from B are all parallel and the force of attraction between B and M is independent of the distance between them when this distance is small in comparison with the diameter of B.

M is a disc that may be moved up or down by means of a micrometer screw. Now when B and G are at one potential and M at some other there will be an attraction which causes B to descend. This force could be measured

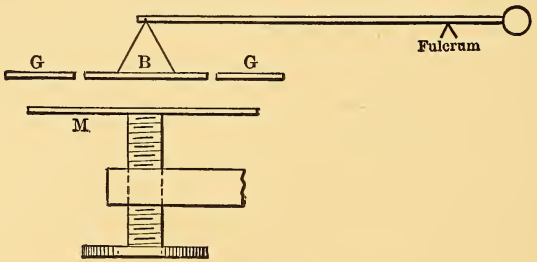

FIG. 44.

by counter-balancing B but better by moving M away till B returns to its normal plane. This does not negate the statement previously made that for parallel lines the force is independent of the distance, because moving M changes the electrostatic capacity of the combination, hence changes the quantity of charge for a given potential, which in turn changes the number of lines of force or the attractive force.

In the practical forms of these instruments, one of which is shown in Fig. 45, many refinements are introduced, chief of which is an arrangement of lenses by which the level of

B can be accurately determined. The *absolute electrometer* differs from the attracted-disc electrometer only in that the counterweight is replaced by an arrangement of springs, whereby the attractive force may be measured in dynes.

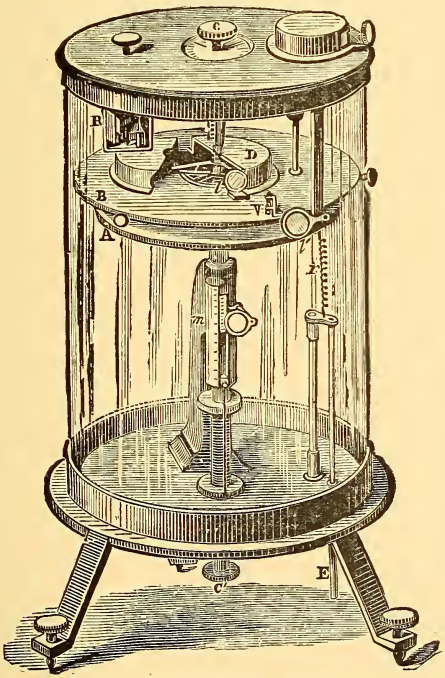

FIG. 45.

Lord Kelvin describes two methods of using the instrument. The *idiostatic* method consists in keeping M and B at the potential difference to be measured (G being at the same potential as B in every case) and adjusting M till B maintains its proper level.

The difference of potential, $V_1 - V_2$ is, according to section 24,

$$V_1 - V_2 = 4 \pi \sigma d.$$

If S be the area of the movable disc, B, $S\sigma$ is the quantity of electricity on it, and again from section 24, it will be attracted to the fixed plate with a force

$$F = 2 \pi \sigma \times S \sigma$$

From which

$$\sigma = \sqrt{\frac{F}{2 \pi S}}$$

Substituting this in the above equation,

$$V_1 - V_2 = d \sqrt{\frac{8 \pi F}{S}}.$$

If the values of d, F, and S are expressed in centimeters, dynes, and square centimeters respectively, then the potential difference will be expressed in absolute electrostatic units, which multiplied by about 300 will become volts. (See Appendix.)

The absolute distance, d, is hard to determine, hence the *hetrostatic* method is employed. G and B are kept constantly charged to a certain potential, and M is connected successively with the two points whose potential difference is to be determined. If V_1 and V_2 be the potentials of the two points, and d_1 and d_2 the respective distances which bring B to a balance, then the required potential difference

$$V_1 - V_2 = (d_1 - d_2) \sqrt{\frac{8 \pi F}{S}}.$$

$d_1 - d_2$ being merely the difference in micrometer readings it is easily obtained.

43. Quadrant Electrometers.— The principle of Kelvin's quadrant electrometer is shown in Fig. 46. Four brass

quadrants are arranged as shown in a horizontal plane. Small air gaps are left between them and the alternate quadrants are in electrical connection. Just above the quadrants is a centrally suspended vane of thin aluminum, shaped as shown. Although it is neither magnetic nor needle-shaped this vane is frequently called the needle of the instrument, because of its homology to the old type galvanometer needle. When used idiostatically, one pair

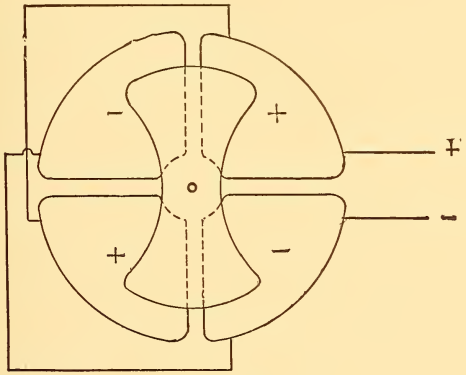

FIG. 46.

of quadrants and the vane are kept at one potential and the other pair at some other potential, usually that of the earth. The first pair will repel the vane and the second attract it, owing to the arrangement of the charges, and the angular displacement may be made a measure of the difference of potential. This arrangement will only measure potential differences of 30 volts or more. The hetrostatic method is much more delicate and is more frequently used. If $V_1 - V_2$ is the difference to be measured, one pair of quad-

rants is charged to V_1, the other pair to V_2, and the vane to V_3, V_3 being much greater than V_1 or V_2. If now c be the capacity per unit angle of the condenser formed by a portion of the vane well within the quadrant and away from the vane's edges, and the vane's deflection is increased by $d\theta$, so that an angular portion $d\theta$ passes from each quadrant of one pair to the quadrants of the other pair; then the corresponding increment of energy, by section 34, is

$$dW = \tfrac{1}{2} c\, d\theta\, (V_3 - V_2)^2 - \tfrac{1}{2} c\, d\theta\, (V_3 - V_1)^2$$

or

$$\frac{dW}{d\theta} = c\,(V_1 - V_2)\left(V_3 - \frac{V_1 + V_2}{2}\right).$$

Now $\frac{dW}{d\theta}$ is the moment of the force producing the deflection θ and must be equal to the moment of torsion resisting the deflection. The moment of torsion in either mono- or bifilar suspension may be considered proportional to the angle, for small angles, hence

$$\frac{dW}{d\theta} = c_1\,\theta.$$

Furthermore V_3 is usually so great that the expression $V_3 - \frac{V_1 + V_2}{2}$ may be considered constant, so we can let

$$V_3 - \frac{V_1 + V_2}{2} = c_2.$$

Substituting these values, we have

$$c_1\,\theta = c\,(V_1 - V_2)\,c_2$$

or, letting $\frac{c_1}{c\,c_2} = m$

$$V_1 - V_2 = m\,\theta.$$

m is the constant of the instrument and may be determined by using a known difference of potential for $V_1 - V_2$ and observing the corresponding θ.

In the practical instrument many refinements are introduced. Bifilar suspension is used. Two sets of quadrants are arranged one above, the other below the vane,

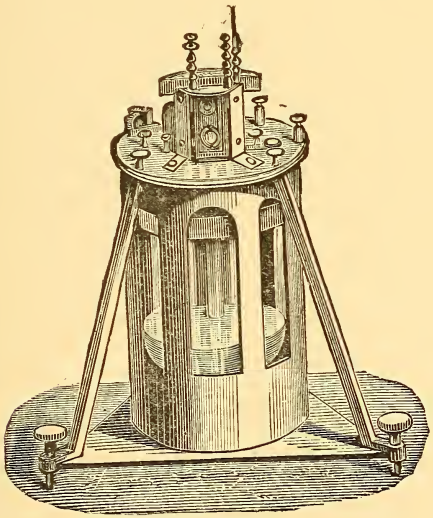

FIG. 47.

the peripheral edges of the corresponding members of the upper and lower set being united, forming a sort of cylindrical box around the vane. Readings are taken by a beam of light reflected along a scale by a vertical mirror attached to the vane. The whole is mounted in a glass vessel the outer side of which is coated part way up with tinfoil. Concentrated sulfuric acid is inside and the combination

L. of C.

forms a Leyden jar, which may be highly charged to a potential V_3 and then made to keep the vane at this potential. The sulfuric acid also serves to keep the atmosphere within the jar very dry. An auxiliary attracted-disc electrometer is mounted on the apparatus and is called the *gauge*. It serves to indicate the potential of the Leyden jar V_3. A circular level and levelling screws are provided. Fig. 47 shows Kelvin's quadrant electrometer in its commercial form.

Prof. H. J. Ryan has designed a quadrant electrometer of unique form, shown in Fig. 48. The quadrants are set in quartz discs, which in turn are mounted on the two circular metal cover plates, and are faced with mica to prevent short circuiting the instrument should the vane get out of place and touch them. The cover plates are held in position by spring clips, three pegs being provided over which each cover fits, insuring that the quadrants always maintain the same positions. Opposite quadrants are automatically connected on assembling the instrument by metallic strips which make contact with binding posts mounted on the quartz discs and appearing on the outside of the cover plates. The "needle" in this case is an aluminum cylinder with quadrangular portions cut away. A mirror is mounted upon it so that its silvered surface lies in the axis of rotation. The cylinder may be suspended by a metallic fiber and charged through it, or by silk or quartz fiber, in which case the connection is made by means of a very fine platinum wire, running loosely from the cylinder to a hook, which latter is connected with an external insulated binding post. The method of making this fine wire is of interest. A very fine platinum wire is heavily silver-plated. The wire thus formed is drawn out as fine

as might be and the silver then dissolved off by suitable acids. The insoluble platinum core is then found to be a wire only .0002 inch (.2 mil) in diameter! Rigidly connected to the aluminum cylinder, and exactly in the

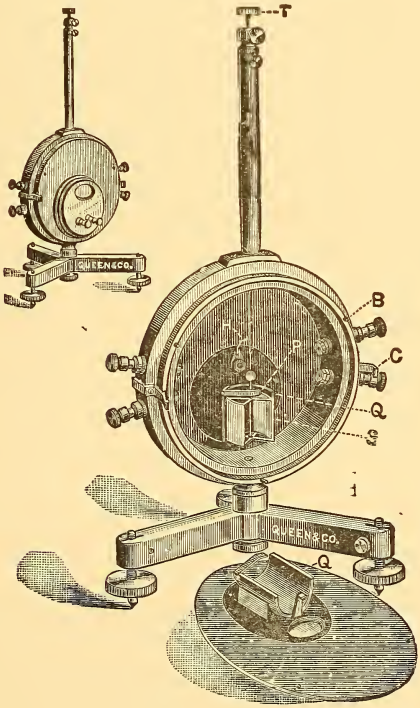

Fig. 48.

center of the containing case is a small magnetic needle. On the outside of the case and symmetrically disposed with reference to the needle are two coils of insulated wire of different numbers of turns, whose terminals are insul-

ated binding posts. The instrument may be used in two ways: either the E. M. F. or potential difference may be calculated by a deflection method as in the ordinary quadrant electrometer; or a zero method may be employed, a measured current being passed through either or both of the coils, acting on the needle and bringing the system to zero. Thus the voltage can be measured by a properly calibrated ammeter in the restoring circuit. By this method potentials as high as 5,000 volts may be measured.

Prof. H. S. Carhart has devised a variation of the quadrant electrometer which uses a zero method, the returning force being supplied by applying torsion to the suspension, exactly as in the electrodynamometer. The cylindrical box-like set of quadrants of the Kelvin instrument is replaced by two similar boxes mounted coaxally and consisting of semicircles instead of quadrants. The two vanes are of aluminum and are also semicircular. They are mounted diametrically opposite on a rigid aluminum wire, and at such a distance apart that one may play freely in each box. The suspension is a phosphor-bronze wire .0015 inch in diameter, terminating in a hard rubber torsion head. A pointer attached thereto plays over a graduated, completely circular scale. A platinum-silver wire spiral connects the lower part of the "needle" to a pin at the bottom of a cup of paraffin oil. The oil acts as a damper and the whole coil serves to conduct the charge to the "needle." A mirror is mounted on the wire connecting the two semicircular vanes. The opposite portions of the two boxes are cross-connected and attached to external binding posts. The vanes are also connected to one of the binding posts, compelling the use of the instrument idiostatically. When the instrument is charged the "needle"

deflects, is brought back to zero by turning the torsion head, and the angle turned through is read from the scale. A calibration curve will give the impressed pressure in volts.

Aryton and Mather's zero electrometer is similar to the one last described, in so far as the returning force is concerned, but differs in that the deflecting force is due to attraction, not repulsion. The "needle" is a rectangle of aluminum ribbon, through the centers of the short sides of which is passed a rigid axis of the same material. The axis is suspended from above by a phosphor-bronze fibre terminating in a torsion head with index and scale. To the lower end of the axis a light pointer is attached at

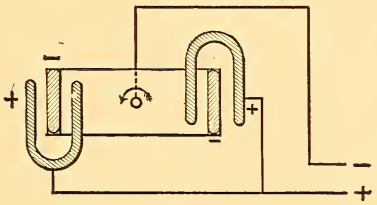

Fig. 49.

right angles which plays over a zero mark. Parallax is avoided by placing a mirror under the needle, enabling the observer to determine when he is normally above it. Around each vertical side of the "needle" is bent a strip of metal termed an *inductor*. An axially normal section of the "needle" and inductors would appear as in Fig. 49. The inductors are connected to one binding post and the "needle" to another. Any difference of potential set up between them will give the "needle" a tendency to swing further into the inductors. This is countered by turning the torsion head till the pointer is on zero, when

the scale at the head can be read and the required value determined from a calibration curve.

All the instruments described above have the advantage that they require no current, wherein they differ from galvanometers. Hence they are adapted to electrostatic measurements where, as a rule, no current is to be had, for electricity in *motion* cannot be part of a *static* phenomenon. Furthermore their capacities are so small as not to materially reduce the charge of any moderately charged body, and for the same reason, when used for alternating current measurements, they take almost no charging current. On direct currents they take absolutely no power. Since their indications are independent of any electrical resistance within them they are independent of temperature variations.

44. Capillary Electrometers.— The principle of the capillary electrometer was first stated by Lipmann, but the instrument has never attained the wide popularity of those previously described. This principle is the change of surface tension between mercury and an electrolyte (dilute sulfuric acid generally) when the difference of potential at the point of contact changes. The instrument consists of two quantities of mercury electrically connected by sulfuric acid, the contact surface at one quantity being comparatively large, that at the other very small, in a capillary tube in fact. If now the smaller surface assumes a position of equilibrium due to the different heads, capillarity, and surface tension, then a difference of potential between the two liquids will cause the meniscus to move to some other position. The effect of a potential difference between the two quantities of mercury will practically all be felt at the smaller surface.

A simple laboratory form of instrument is shown in Fig. 50, which is practically self-explanatory. Under the capillary tube *T* is a finely divided scale, which with the aid of a lens allows small deviations of the meniscus to be noted. The whole is mounted on an inclined adjustable board, so that the position of equlibrium can be regulated more easily than by removing or adding mercury to *A*. The wire that leads to *B* must be insulated from the sulfuric acid, preferably by sealing in a glass tube. The capillary is about .5 mm. bore, and the acid is diluted 1

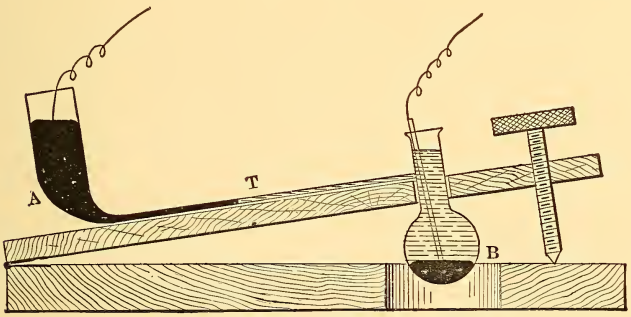

FIG. 50.

to 6 by volume. The delicacy of the electrometer is less the more inclined the capillary is. When horizontal an accuracy of .001 volt may be attained. The faults of this instrument will be discussed later.

A much more practical type is shown in Fig. 51. This is as nearly portable as an instrument comprising three liquids and glass parts can be. *T* is a thick test tube having a platinum terminal fused in its lower end. This terminal is covered with a layer of mercury and the rest of the tube is nearly filled with dilute sulfuric acid. The

top of T is closed by a tight-fitting rubber cork, through which projects the capillary tube C, made by drawing out a thermometer tube. This must rest close against the side of T to avoid troublesome refractive effects. A piece of flexible rubber tube B is fitted over the top of C and wrapped to prevent leakage. This is spanned by a screw

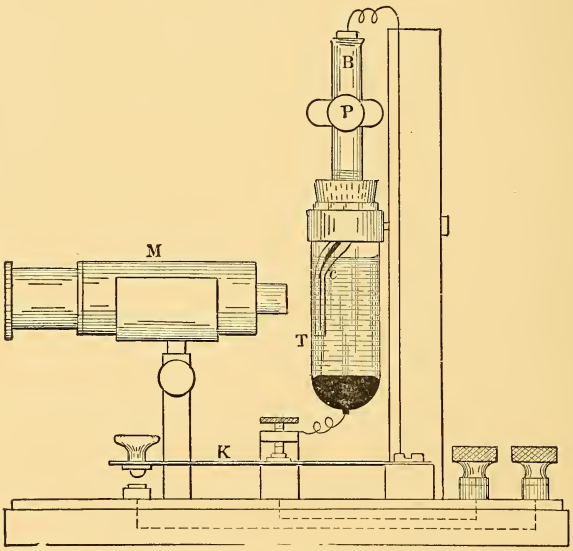

FIG. 51.

pinch-cock P and is filled with pure mercury. Its upper end is closed by a piece of glass into which is fused the second platinum terminal. On compressing P the mercury is forced through C, and on slightly releasing the pressure it draws part way up, bringing up the acid after it. The whole is now ready for use. The microspope M fitted with a micrometer eye-piece may now be adjusted to bring

the meniscus to the hair line. The key K serves to keep the instrument short-circuited save when in actual use, a very necessary condition. Mr. C. F. Burgess, who described this instrument before the American Institute of Electrical Engineers, claims for it a sensible deflection for .001 volt and suggests the possibility of ten times that sensitiveness when certain refinements are observed.

This instrument cannot be used quantitatively, because: equality of bore is impracticable, the deflection is not necessarily proportional to the potential difference, it is impracticable to maintain always the same density of acid, and most important of all, the instrument cannot be used on pressures exceeding one volt, for then electrolysis sets in and the readings become worthless. The mercury in the capillary should suffer cathodic polarisation, *i. e.,* be connected to the negative side, if any attempt be made to use it quantitatively. Cathodic polarisation of over one volt causes the formation of hydrogen bubbles, while anodic polarisation of over a few hundredths of a volt causes the mercury to become impure and the meniscus to lose its mobility.

All the facts indicate a usefulness for the capillary electrometer only as a zero instrument in some compensation method. While its accuracy is not so great as that of the finest galvanometers, it may be used where they cannot because it is only slightly sensitive to mechanical, almost indifferent to electric, and completely indifferent to magnetic disturbances. Furthermore, it is practically dead beat.

It is inevitable that the capillary electrometer will be overpolarised by accident when in operation, and then the one last described has a great advantage over the

simple form first described. In the first arrangement it would cost some pains to remove a bubble or drop of mercury from the capillary, while in the second instrument

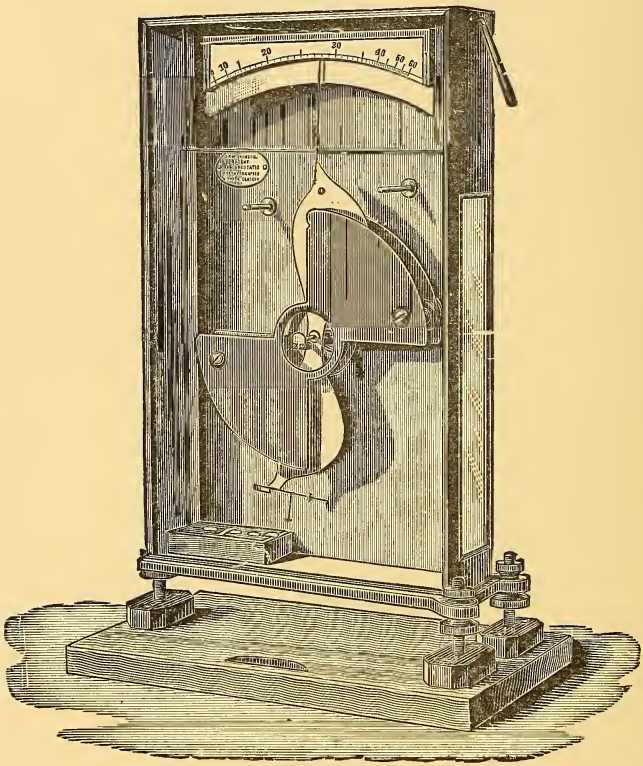

Fig. 52.

it is merely necessary to compress the pinch-cock till a drop of mercury falls from the capillary into the bottom of T when the instrument is in its original condition.

45. Electrostatic Voltmeters.—Any non-zero electrometer may be calibrated to read directly in volts, thus becoming a voltmeter. A few of the more general forms will be here shown. Fig. 52 shows Kelvin's vertical

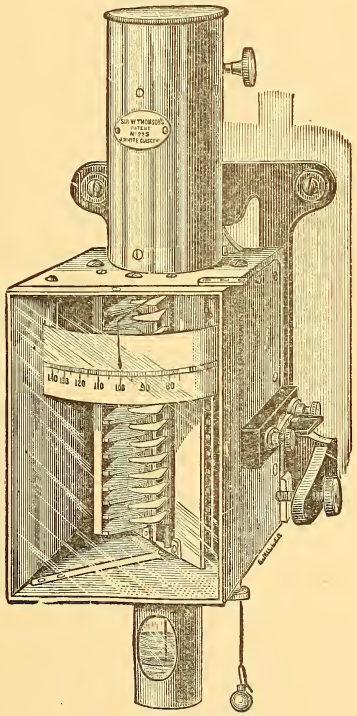

Fig. 53.

electrostatic voltmeter. As may be clearly seen it consists of a vertically arranged quadrant electrometer with one pair of quadrants deleted and used idiostatically. Instead of a mirror it has a pointer playing over a scale graduated directly in volts. These instruments may be

made to operate on from 400 to 20,000 volts, any one instrument having for its maximum reading ten times its minimum reading; for instance, from 400 to 4,000 volts, or from 2,000 to 20,000 volts. These ratings are for direct current, and should be just about halved for alternating potentials, because of the greater sparking distance of the latter for a given effective voltage.

For great accuracy and permanency, such as is required of voltmeters for ultimate standards in laboratory work, Kelvin's multicellular voltmeter may be used. These are made with either vertical or horizontal scales. As shown in Fig. 53, they consist of a number of modified quadrant electrometers, arranged coaxally, the vanes all being connected together. This gives considerable torque and allows substantial and accurate construction. The ranges of these instruments are from a minimum to a little over twice that for a maximum, and they can be made to operate from 30 to 2,400 volts.

The instruments thus far described have been laboratory instruments. A type of commercial switchboard electrostatic voltmeter is shown in detail in Fig. 54. This is the Stanley static station voltmeter. The principle is again that of the quadrant electrometer. The vanes *A A* are partial cylinders, as are also the fixed quadrants. The instrument is idiostatic, the vanes *A A* and the quadrants *B B* being all connected to one side of the circuit, while *C C* are connected to the other. The movable member is mounted on knife-edge jewels and is controlled by the spiral spring *S*. Two fans, *F F*, attached to the vane and inclosed in the damping boxes *D D* make the instrument dead beat. These voltmeters range from 750 to 1,400 volts, and from 1,400 to 2,800 volts.

The advantages of this type of instruments for switchboard use are: permanence of calibration, quick action, no loss of current, requires no step-down transformer, free from effects of frequency or wave shape, unaffected by external fields, and has no temperature error.

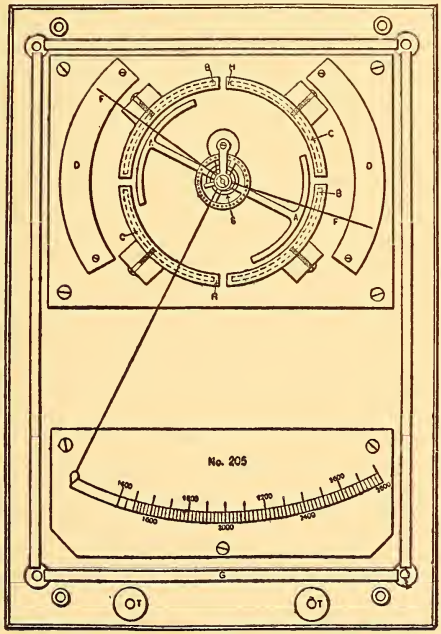

FIG. 54.

While the use of static instruments is in most respects far preferable to that of electromagnetic instruments on high tension service, the former are open to one serious objection, and that is the tendency of the parts to discharge across the air gap between them, giving rise to sparks det-

rimental to the instruments. By making the gaps wide enough to prevent the possibility of this happening, the sensibility of the instrument is seriously impaired. To overcome this trouble Prof. Elihu Thomson has devised a method of inclosing the instrument in a glass tube or bulb, exhausted to an extremely high Crookes vacuum. By this means the sparking distance for a given voltage is greatly reduced, thereby increasing the sensibility and decreasing the delicacy of the instrument. Fig. 55 shows such an arrangement.

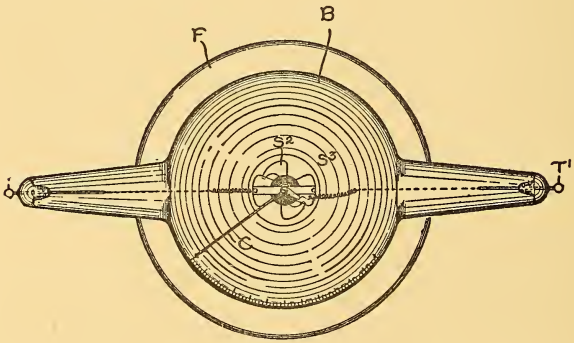

FIG. 55.

46. Electrostatic Ground Detectors.— The electrostatic electrometer principle may be applied to the detection of grounds on transmission lines. Fig. 56 shows the Stanley static ground detector. This is essentially a differential static voltmeter, composed of six fixed metal vanes, two primary and four secondary, and a movable vane of sheet aluminum carrying a pointer. The movable vane and pointer are mounted on a shaft and held centrally between the fixed secondary vanes by two sapphire jewel

bearings, the whole being controlled by a spring. The diagonally opposite fixed secondary vanes are connected, and each pair is indirectly charged (statically) from one of the fixed primary vanes, which in turn receive their

Fig. 56.

charges from the line wires. The movable vane is permanently connected to earth, and is inductively acted upon by each pair of fixed secondary vanes. The force of each

pair being equal and opposite the pointer takes a medial position. This instrument is thus hetrostatic in its operation. If, however, one line wire becomes grounded the movable vane receives a charge the same as that of the fixed vanes belonging to that wire, and the arrangement becomes idiostatic and the movable vane will have a ten-

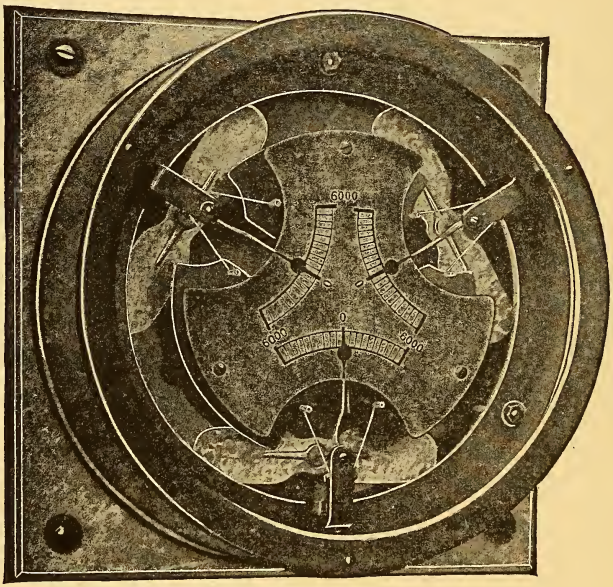

Fig. 57.

dency to lie under the other pair of fixed vanes. This may be made to indicate not only a ground, but on which wire such ground is. Since there is no electrical connection between the line wires and the instrument save to the two primary fixed vanes, and as these are imbedded in the base of the instrument, it would be impossible, no mat-

ter how badly the instrument was out of order, for a short circuit to occur between the lines through the instrument.

The General Electric Company have devised a ground detector for use on three phase circuits. Fig. 57 gives a view of this instrument, and Fig. 58 shows the electrical

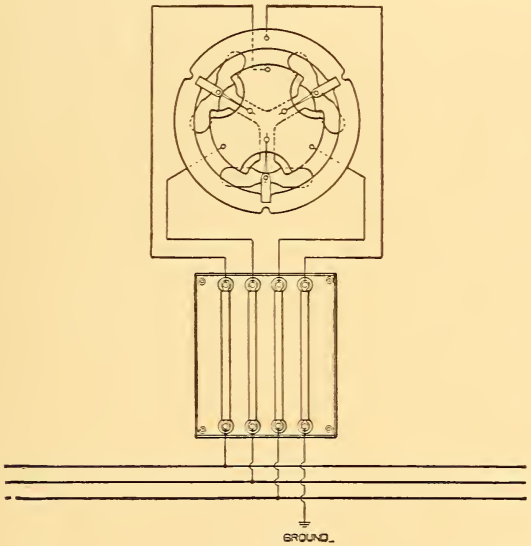

FIG. 58.

connections. Here there are three fixed segments corresponding to the quadrants of previous instruments, and three aluminum movable vanes, with pointers attached, arranged as shown. Light counterweights are used to hold the pointers normally at the zeros. Each of the fixed segments is connected to a line wire and all the movable vanes are connected to the ground, as shown in Fig. 59. When

no ground exists the needles point toward the center. When a ground occurs on one of the lines the two adjacent needles are deflected toward the segment connected to that line. Should a ground occur on two lines simultaneously, the needle between the segments belonging to the grounded lines will point to the line having the least resistance ground, and the remaining two will be deflected toward the grounded segments. If a vane became displaced and touched a segment it would in itself constitute a ground, and if it touched two segments at once, it would short-circuit the lines and ruin the instrument. To guard against this contingency the segments are connected to the lines only through high carbon resistances, as are also the vanes to the earth. These are shown in Fig. 58.

CHAPTER VI.

HIGH POTENTIAL STATIC GENERATORS.

47. Friction Machines.—It was but natural for the early experimenters to seek for some more satisfactory means of securing electricity than the laborious process of rubbing a glass or vulcanite rod with cloth or fur, which process was slow, and by its very nature intermittent. As might be expected the first step was to devise a means whereby the rubbing might be mechanically and continuously performed.

The most primitive machine for this purpose was probably that attributed to von Guericke, which consisted merely of a sphere of sulphur mounted on a spindle, with means provided for revolving it. If the hand were held against this sphere while in rotation, a slight charge would be imparted to the body. Succeeding steps replaced the sulphur sphere with one of glass, substituted a leathern cushion for the hand as a rubber, simplified the construction by changing the spherical revolving member to a cylindrical one, and added a loose metal chain which, hanging against one side of the revolving cylinder, could conduct the charge away.

A practical form of the friction machine used to a limited extent at present in laboratory demonstration consists of a horizontal glass cylinder, mounted on metal shaft, equipped with crank and handle, and suitably mounted in bearings on strong supports. On one side of the cylinder, and pressing against its convex surface, is a pad of leather,

covered with a suitable amalgam, and having an apron or flap of silk attached, which lies over the upper half of the cylinder. The rubbing pad is usually mounted on an insulating support, but if it is never desired to collect the negative charge which accumulates thereon, this refinement may be omitted. The direction of rotation of the glass cylinder is such as to draw the silk apron the more tightly to the surface, thus considerably increasing the area exposed to contact, and increasing the amount of charge. On the side of the cylinder opposite the rubber is arranged a horizontal metal comb with the teeth pointing toward the axis of the cylinder. The back of this comb feeds into a large metal sphere or round ended cylinder, which is customarily called the "prime conductor."

The action of the comb is this: The friction, or more properly the contact between the rubber and the glass, causes electrical action such that a negative charge accumulates on the amalgam-coated surface of the rubber and a positive charge on the glass. The surface of the glass in revolving brings this charge underneath the comb, where it acts by influence on the prime conductor, attracting a negative charge to the teeth of the comb, and repelling a positive charge to the further extremity of the prime conductor. Now, as stated in section 14, the pointed teeth of the comb are but poorly adapted to retaining a charge, and the negative charge escapes from them in the form of an electric wind which blows directly on the cylinder. The negative electricity thus carried to the surface of the cylinder unites with the positive charge already there, leaving the surface approximately neutral as it approaches the rubber for a repetition of the cycle. Thus increments of free positive charge are being added to the prime con-

ductor as elements of the surface pass the comb. The quantity of electricity that may thus be accumulated in the prime conductor depends upon the insulation of the latter. For the electrostatic capacity of the prime conductor and its insulation from surrounding objects are sensibly constant, hence, as the amount of charge increases, so also must the potential increase. But the leakage current, which follows Ohm's law, will increase as the potential increases; so it is evident that a point will be reached where the leakage current carries away from the prime conductor as much electricity as the revolving cylinder brings to it. This leakage of electricity occurs not only over the surface and through the substance of the insulating supports of the prime conductor, but a passage of electricity from the conductor directly into the air itself occurs, which gives rise to the "brush discharge," which is treated of in a subsequent paragraph. In using this instrument the rubber, which because of its amalgam coating is a good conductor, is generally connected to ground by a chain or other convenient means, to carry off the negative charge which tends to accumulate thereon, thus leaving it in a condition to more strongly excite the electric action.

This amalgam with which the rubber is coated serves a double purpose. As just mentioned, it renders the rubbing pad much more conductive than it would be if simply covered with leather, and it offers as a friction surface a substance between which and glass more vigorous electric action takes place than between leather and glass. In the "Electrostatic Series," given in section 6, it is seen that the interval between glass and metals is greater than that between glass and the hand, the latter being, electrostatically, about the same as leather. A simple amalgam of tin and

mercury will answer the purpose. A more satisfactory compound, known as Kienmayer's amalgam, consists of equal parts by weight of tin and zinc melted together, to which, while still molten, is added twice their weight of mercury. These amalgams are usually applied to the rubbing cushions through the agency of some thick grease.

A more usual form of the friction machine has for the revolving element a circular disc of glass instead of a cylinder. This form of construction, besides being much cheaper, since a perfect glass cylinder is difficult to make, allows the use of two or more sets of rubbers and allows both sides of the glass to be used, thus greatly increasing the surface exposed to friction without increasing the size or weight of the machine. Furthermore, the distance over the surface between charges of opposite sign is greater, tending to reduce losses by internal discharge, or leakage over the surface of the revolving member. Friction machines have been entirely superseded for all but laboratory and demonstration purposes by the influence machine, the simplest form of which is the electrophorous, described in section 10.

48. The Toepler Machine.— Toepler began the experimental construction of influence machines about 1865, before which date little had been done along this line, though the first so-called " doubler " was invented in 1786. A doubler is simply an elaboration of the electrophorus (§ 10) such that the charge influenced on the cover is not discharged to earth but to a prime conductor. The cover is then sent back to the electrified hard rubber, charged by influence again, and this second charge discharged into the prime conductor. As this operation is repeated it is evident that the quantity of electricity on the prime con-

ductor will be increased, and at each increase its potential will also be raised.

The Toepler machine, as now constructed, is shown in sketch in Fig. 59. A fixed vertical glass plate, usually circular for convenience, has a hole in its center through which a horizontal revolving axis can work between bearings, one on each side of the fixed plate. Mounted on the

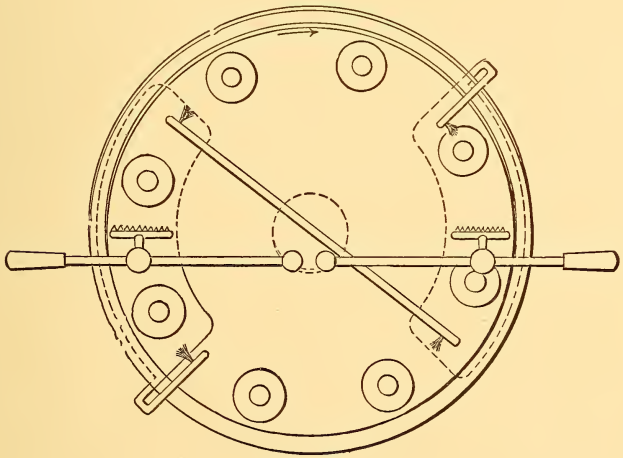

Fig. 59.

axis in front of the fixed plate is a revolving glass plate of a diameter somewhat less than that of the fixed plate. The distance of the revolving plate from the fixed plate is made as small as is practicable. On the front of the rotating plate, near the periphery and at equal intervals, are pasted small circular discs of tinfoil, called *carriers,* each disc having at its center a small protruding metallic

button. This button allows of metallic connection between the moving carriers and the fixed brushes without unduly wearing the former. On the back of the stationary plate and at opposite ends of its horizontal diameter are fixed two pieces of paper, called *field-plates* or *inductors.* These field-plates are of somewhat segmental shape, as shown by the dotted lines in the figure, and are made of paper, because it is found that this substance, being a fair conductor of static charges, is more satisfactory than a good conductor, such as tinfoil. As shown in the figure, from the end of each field-plate which the moving carriers first approach, a conducting arm is bent so as to hold a soft wire brush over the front of the revolving plate in such position that the brush just makes contact with the metal buttons on the carriers as the latter move under the former. These are called the *appropriating brushes.* Two more brushes held in place and in electric connection by a diagonal metallic bar, called the *neutralizing rod,* touch the buttons just as the carriers are passing beyond the end of the field-plates. In front of the rotating plate and opposite the centers of the field-plates are two metallic combs, teeth pointing to the plate, and connected to sliding discharging rods, called *electrodes,* the ends of which can be separated by any required distance.

The action of the machine is as follows: Neglect for the moment the discharging combs and rods. Suppose a small negative charge to rest on the left hand field-plate. This will attract a small positive charge on the under side of a carrier that happens to be over it. A similar negative charge will be repelled to the front side of this carrier. Now assume the disc is revolved in the proper direction. As the carrier passes under the neutralizing brush it is

still under the influence of the field-plate, therefore the negative charge on the front of the carriers is repelled into the neutralizing rod and disposed of, as will be shown later. As the carrier passes beyond the influence of the field-plate the positive charge bound on the under side of it will become free and, as the corresponding negative charge has been disposed of by the neutralizing rod, the carrier will have a free positive charge overlying its whole surface. When the carrier passes under the next brush a part of this charge will be communicated to the right hand field-plate, thus giving it a charge of opposite sign to that of the left hand field-plate. The carrier now undergoes the second half of the cycle, being charged and discharged as in the first half, save that all the signs are changed, a positive charge being sent out into the neutralizing rod. This charge unites with an equal negative charge coming in from the left hand end, as mentioned before, and thus the rod is kept neutral without the necessity for a ground connection. The carrier is now on its way toward the left field-plate with a free negative charge, a part of which it gives up to the plate as it passes under the next brush, thus increasing the original charge. This cycle is repeated for each carrier at each revolution, and it is thus clear that the field-plates will soon be charged to so high a potential that the amount of electricity lost from the field-plates by leakage equals that brought to them by the carriers. The function of the metallic carriers and the brushes is simply to keep the field-plates as highly charged as possible.

Now consider the discharging combs. Imagine the ends of the discharging rods to be in contact. While the machine is building up these combs and rods have no func-

tion. When, however, the field-plates have their potential raised to a certain point, their influence on the metal of the discharging apparatus becomes so strong — the two intervening plates of glass offering no obstacle — that the electricity attracted to the points of the combs can no longer rest on them but is torn off in the form either of an electric wind (§ 14) or a brush discharge. The electricity thus transferred from the combs rests on the front surface of the revolving plate, and is not dissipated to any extent, as the plate is a non-conductor. Now positive electricity is fixed on the rotating plate above the left hand or negative field-plate. As the machine revolves this electricity is brought around under the right hand comb, from which negative electricity is streaming. This latter unites with the positive already on the rotating plate, leaving the plate approximately neutral and in condition to repeat the process. This constant discharge from the combs implies a transference of electricity along the electrodes, and this is the fact. The circuit is curious in that it is not a complete conducting circuit, but through one part of it the electricity is carried by mechanical means. Beginning at the face of the revolving plate, just under the first comb, the circuit can be traced as an ordinary conducting circuit up through the brush discharge, through the comb and electrode, through the other electrode, comb, and brush discharge to the face of the plate under the second comb. Here the circuit ceases to be conducting, the electricity being carried back to the starting point by the rotation of the plate. It is in this part of the circuit that the energy is supplied that keeps the electricity moving and does the work that is expended in sparks or vacuum tubes or other apparatus that may be put

in circuit. The action of this circuit under various conditions of resistance and capacity are treated of in a subsequent paragraph.

This machine, or slight modifications of it, are sometimes called Toepler-Holtz or Voss machines, because of modifications made by others than Toepler himself. As the inventor left the machine it differed from the one just described, in that the fixed plate consisted of two separate pieces of glass, one to the right and one to the left. The space between these was sufficient to accommodate the revolving axis. In the original machine also the neutralizing bar and brushes were lacking, and the appropriating brushes were fixed at the trailing ends rather than the leading ends of the field-plates. The field-plates themselves were of tinfoil. With this arrangement the charges repelled to the front surfaces of the carriers were added to those on the repelling field-plates. This left the two halves of the machine independent save for the discharge circuit, a condition which lends itself readily to unbalancing the system. It is found that the maximum effect can be obtained from a machine when the field-plates are charged to equal potentials of opposite sign. Toepler found that his machine operated much better if he removed one tooth from each of the combs and substituted a brush that would touch the carriers. The effect of this was to equalize the potentials. It was almost essential that this machine be started with the discharge rods in contact, to aid in the building up, while in the improved form it is somewhat better to have the rods widely separated.

The Toepler machine is very satisfactory, in that it can be started in any weather and without excitation, it being always the case that one field-plate is at a slightly higher

potential than the other. Even if it were not the mere agitation of the air in the neighborhood would cause enough difference to enable the machine to start.

49. The Holtz Machine.— This machine, which is perhaps the best known of the influence machines, consists of a fixed plate and a revolving plate as shown in Fig. 60. The former is slightly the larger in diameter and is pierced by three holes, a circular hole in the center to accommodate the revolving axis, and two "windows" near the opposite ends of a diameter and of a shape somewhat as shown. These windows are sometimes replaced by deep notches

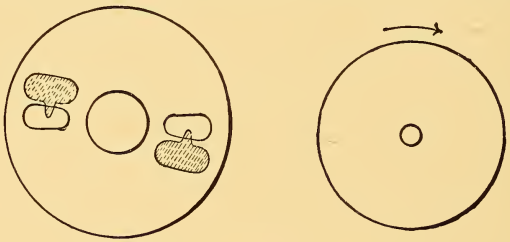

FIG. 60.

running in from the circumference, a form that seems equally good and is much easier to cut. On the back of the fixed plate and at the side of each window are pasted the field-plates. These field plates are usually of paper and are of such circumferential length as to subtend from 10° to 60° of central angle, between these limits their length seeming to make little difference in the operation of the machine. One, or sometimes more, rather blunt-pointed projections from each of the field-plates are brought through the windows and bent so as to point toward but not touch the revolving plate. The revolving plate is mounted concentric with the fixed plate and as

near to it as is practicable, the less the clearance being the better the operation of the machine. There are no carriers on the revolving member, this being simply a shellaced or lacquered glass disc. As shown in Fig. 61 two collecting combs are mounted in front of the machine, one opposite each field-plate, with teeth pointing toward the revolving plate and set a little way back from the edge of the window.

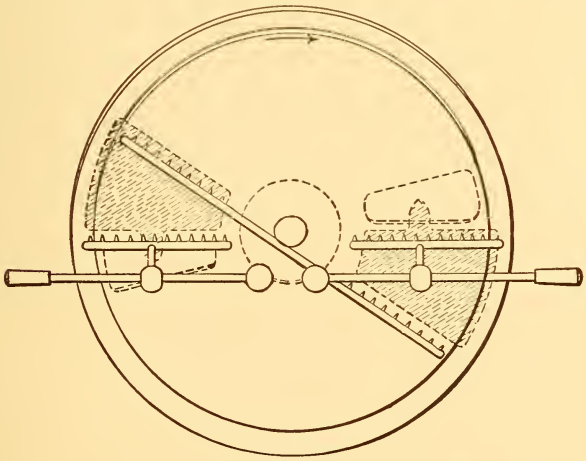

FIG. 61.

It is stated that with these combs set considerably further back from the edges of the windows tinfoil field-plates can be satisfactorily used. These combs are connected to the usual discharging rods or electrodes. This equipment completed the theoretical construction of Holtz's original machine, but he subsequently added, and all modern machines are equipped with, a neutralizing rod fitted with a comb at each end, the rod being held diagonally in such a position that the combs are about over the hinder or trail-

ing edges of the field-plates. The direction of rotation of the moving plate must be toward the projecting points of the field-plates.

The operation of the machine is as follows: The electrodes are put in contact and the disc is rapidly rotated. A charge of some magnitude is then imparted to one of the field-plates. This is usually accomplished by briskly rubbing a sheet of paper with a cloth or fur and then placing it on the back of the fixed plate over one of the field-plates, where it will stay of itself if it is sufficiently well charged to start the machine. Say this negative charge is imparted to the left hand field-plate. This charge repels a negative charge in the collecting apparatus and attracts a positive charge to the teeth of the comb. The charge on the field-plate must be sufficiently strong to tear this positive electricity off from the comb in the form of an electric wind, or the machine will not build up. If the wind does blow upon the revolving plate the front surface of the latter becomes positively charged, and as the plate revolves this charge is brought around over the projecting point of the right hand field-plate. Here it attracts negative electricity from the right hand field-plate which, blowing off the point, becomes bound on the under side of the revolving plate, the glass of the latter acting as the dielectric of a condenser separating two mutually attracting charges. The loss of this negative electricity leaves the right hand field-plate positively charged. This plate now draws a negative wind from the right comb which, uniting with the positive charge already on the front of the revolving plate, neutralizes the latter, thus freeing the negative charge on the under side of the revolving plate, and then leaves an excess of negative electricity on the upper side as well.

As the plate now approaches the left hand field-plate, it is strongly charged on both sides with negative electricity. These charges will draw positive electricity from the left field-plate, neutralizing the under side of the rotating member and leaving the machine in condition to repeat the cycle, the intensity of the charges becoming greater at each repetition until the limit set by the leakage of the machine is reached.

It will be noted that in the above discussion the neutralizing rod and combs play no part, the front surface of the revolving plate being regularly neutralized by the transfer of electricity along the electrodes and through the combs. If now a condenser is connected to each electrode — and there is usually one so connected permanently — the outer coatings of the two being in metallic connection, and if the electrodes are separated to such a distance that the machine is not able to force a spark across the gap, then the machine will suffer reversals of charge and finally complete loss of excitation. The explanation is this: So long as the electrodes are in contact or within sparking distance, the transference of electricity through the discharging circuit is sufficient to cause the neutralization of the surface of the revolving plate and to permit the cycles of operation to recur as described above, but as soon as the gap is made too great for the spark to jump the transference ceases, and the machine will operate only until the condensers are as highly charged as they can be at the potential at which the machine is working. Then neutralization ceases and the positive charge which was laid on the revolving plate over the negative field-plate is carried all the way around and attracts the negative charge of the field-plate through the window to itself. This action is aided by the condensers,

which, as soon as the potential of the machine begins to fall, start discharging in a direction opposite to that in which they were charged. The presence of the neutralizing rod, offering as it does a low resistance path irrespective of the position of the electrodes, assures the satisfactory neutralization of the revolving plate under any conditions.

The Holtz machine is somewhat more powerful than the self-starting machines of similar size but is not always so satisfactory because of its dependence on favorable atmospheric conditions. Sometimes it is impossible to start it in the open room at all, and often it is only after a long period of warming, drying, and rubbing that it will work. In the latter cases it seems as if the internal discharges of the machine improve its condition, for after once being started it usually runs without further trouble, and may even be stopped and easily started again within a reasonable time. The only way, however, to insure any certainty of operation is to inclose the whole machine in a glass case as nearly air-tight and dust-proof as is practicable. In this case should be put a shallow dish full of concentrated sulfuric acid to absorb the moisture of the air, and another full of linseed oil which seems to absorb the ozone given off during operation. The presence of ozone lowers the insulation of the air and decreases the efficiency of the machine. Some Holtz machines are even incased in such a manner that the whole machine and the air surrounding it may be warmed by a flame in an iron oven in the lower part of the case.

Several modifications in the construction of these machines are sometimes observed. The possible use of tinfoil field-plates and a notch instead of a window for the point of the field-plate to project through have already been

mentioned. In another arrangement the field-plate is pasted on the back of the stationary plate and a small row of paper points pasted on the front of the stationary plate in a proper position and the two connected together by a strip of paper extending around the edge of the field-plate. Again the whole field-plate is sometimes brought through the window and bent back for a little way along the front face of the stationary plate. The object of this is to avoid the accumulation of a charge of opposite sign on the front of the glass that would tend to weaken the action of the field-plate. In such an arrangement the position of the point or points is the same as before. A machine was constructed in which the tongues of the field-plates projected through small slits instead of large windows. The action of the machine was weak, the explanation advanced being that the ozone formed had not sufficient room in which to escape, and, therefore, lowered the insulation detrimentally. The matter of the size, shape, number of points and material of the field-plates seems to rest on the preferences of the designer of the machine. This much is certain, that if the material be of too good a conductor and the points be too sharp or too many, the field-plates loose their charges too readily, while if too poor a conductor or too blunt, they do not pick up their charges readily enough.

Holtz machines may also be started by holding an electrified rod of glass or vulcanite in such a position that it will act temporarily as a field-plate, or by using two charged Leyden jars. To start the machine by condensers, these should be oppositely charged, the outer coating of one and the inner of the other being positive. The machine is then set rotating with the electrodes widely separated. The knobs of the jars are then connected to the electrodes

respectively and their outer coatings connected together. The jars will at first discharge but the machine is thereby charged to such an extent as to become operative and the jars are then charged to the full capacity of the machine in the same sense that they were at first. If the machine

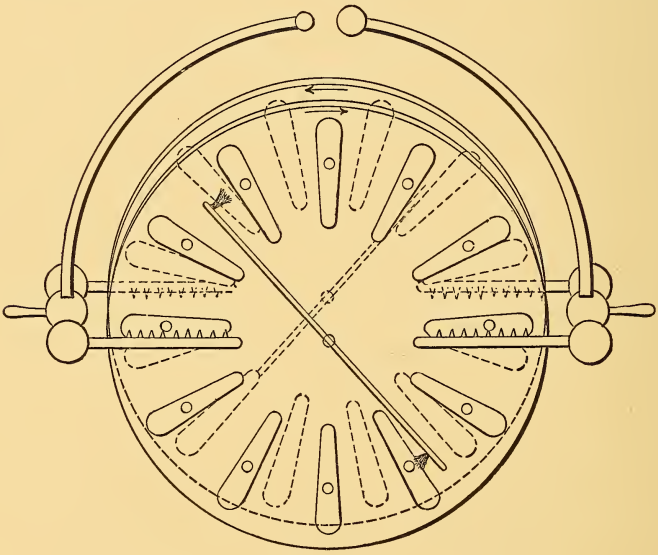

FIG. 62.

were not equipped with a neutralizing rod the electrodes would have to be put in contact immediately after the jars were connected else the whole charge would be lost.

Holtz made many other forms of machine, some with two opposite revolving plates, some that were not regenerative, that is, did not keep their own field-plates charged, but these are now only of historical interest.

50. The Wimshurst Machine.— Unlike the foregoing machines, the Wimshurst has no fixed field-plates. As

shown in the sketch (Fig. 62), two similar, oppositely rotating glass discs are concentrically mounted as near together as is feasible. On the outer face of each of these plates are attached a number of sector-shaped carriers of metal or foil, radially arranged and equally spaced. A neutralizing bar with brushes is provided for each plate, such bar being set at an angle of about 45° from the collecting combs measured in the direction of the rotation of the plate to which it appertains. The collecting combs are bent, horseshoe-like, to allow the teeth to project toward the outer face of each plate and to cover the full radial depth of the metal sectors. The electrodes are customarily mounted on pivots so the distance between them may be conveniently varied. It is usual to have the knobs on the end of the electrodes of unequal size, as it is found that a greater length of spark can be obtained with this arrangement.

The action of this machine should be readily understood because of its similarity to that of the Toepler machine. Consider for a moment the carrier (Fig. 62) that is being touched by the upper front neutralizing brush. In back of this and on the other plate is another carrier which we will assume to carry a small negative charge. This last will act like the first field-plate of the Toepler machine and leave the front carrier with a positive charge after it has passed from under the brush. In the meantime a negative charge will have been repelled down the neutralizing bar, leaving the diametrically opposite carrier on the front plate negatively electrified. As now our original carrier is rotated till it comes opposite the upper back neutralizing brush it will act as a field-plate to the carrier under it on the back plate, thus giving the latter a negative

charge. Meanwhile the negatively charged lower front sector will have influenced a negative charge on a back sector. This process is repeated for each carrier, the tension of the charges increasing at each repetition until the limit set by the leakage of the machine is reached. It will be observed that each plate is carrying positive charges toward the right collecting comb and negative charges toward the left. These combs as it is frequently put "collect" these charges, the more proper statement being that the charges on the carriers attract and are in part neutralized by brush discharges drawn from the combs.

This machine is self-starting in practically any kind of weather if the neutralizing brushes are clean and make good contact simultaneously with diametrically opposite carriers. As the number of sectors is decreased and the space between them correspondingly increased the length of the sparks obtained is increased but their number is decreased. This is a result of the lessened internal leakage due to the wider spacing. If the number of carriers is reduced beyond a certain limit, about six per plate, the machine ceases to be self-exciting but will work well after being properly charged. It is usual to start the machine with the electrodes in contact, but this precaution can be omitted as the discharge circuit plays no necessary part in the upbuilding of the charges on the sectors. Unlike the Holtz and the Toepler, the Wimshurst machine never reverses its polarity, even when the electrodes are separated much beyond the sparking distance. Neither does it loose its charge if left running with the electrodes in contact.

51. Static Machines as Motors.— Any of the machines described in the foregoing sections can be operated as a motor if supplied with electricity of sufficiently high ten-

sion. If one Holtz machine be regularly driven as a generator and its electrodes be connected to the electrodes of a second Holtz machine and the electrodes of both machines be well separated so that sparks do not jump the gaps, the second machine will start up and run in the backward direction, that is, away from the tongues of the field-plates.

Such a combination as this has been operated by an electric motor whose mechanical output at any given electrical input had been previously determined. The mechanical output of the second Holtz machine, running as a motor, was measured by a delicate Prony brake, while the input to the electric motor was measured by a watt-meter. After allowing for the losses in the electric motor, but not allowing for losses in the driving belt, the speed-measuring device nor for any electrical losses between the two static machines, the combination of Holtz generator and Holtz motor showed an efficiency of some 7 per cent. Assuming the machines to have the same efficiency this would indicate an efficiency in each machine of $\sqrt{.07}$ or 27 per cent.

52. The Dropping Generator.— The three machines described in the previous part of this chapter illustrate the three principal types which are the foundation for most of the modern influence machines. In practice many modifications are introduced, one of the principal ones being the running of a number of plates in multiple to secure a greater volume of discharge. Multipling the plates also slightly increases the spark length for the same diameter plates. Some of the salient features described above are, however, generally to be found in all static generators.

A high tension electric influence machine of an entirely

different character is the dropping generator due to Kelvin. This machine which is of more theoretical interest than practical utility is easily described. From a common reservoir two streams of water are led. (Fig. 63.) Each stream is allowed to fall vertically downward through a hollow insulated metal cylinder 3 or 4 inches long and about $1\frac{1}{2}$ inch in diameter. These streams are so adjusted

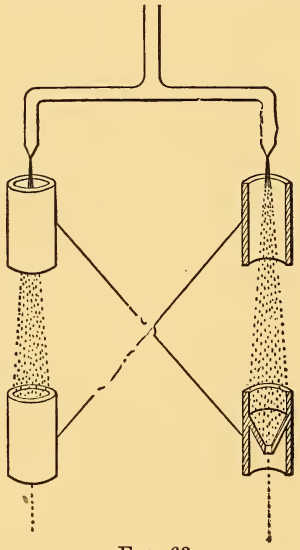

Fig. 63.

that they break into drops when about half-way through the cylinders. The drops are collected beneath the cylinders by insulated metal funnels and the water is allowed to fall in a spray from the funnels to the waste receptacle. As these funnels are to become the oppositely charged parts of the apparatus, a solid stream from them to the waste is impracticable because of the short-circuiting effect.

Let us assume that one cylinder is slightly charged in the negative sense. As the drops that leave it break off from the solid stream they are positively electrified, the corresponding negative charge being repelled up the stream to the reservoir. These drops are collected in the funnel, and as there can be no charge within a conductor, the drops immediately give up their charges which are transferred to the exterior of the funnel. The drops then issue from the bottom of the funnel approximately neutral. This funnel is kept in metallic connection with the second cylinder, thus the latter becomes positively charged. Then the second funnel has added to it constant increments of negative charge. This funnel is likewise connected by wire to the first cylinder, thus augmenting the negative charge thereon. The limit to the increase of potential obtainable by this apparatus is set by the leakage of charge over the surface, as in foregoing cases, but here it is easier to improve the insulation, as the various parts may be mounted on long glass rods or supports, and there are no moving parts save the water. It is stated that 7,000 volts — electromagnetic system — can readily be obtained by this machine.

As the drops issuing from either of the cylinders are highly charged they are strongly self-repellant, and as the funnel beneath waiting to catch them is similarly charged, tending further to repel the drops, it usually happens that only a portion of the water finds its way into the desired receptacle, the rest being scattered throughout an uncomfortably wide range.

53. Discharges of the Static Machine.— The charged terminals of an electric machine or a battery of Leyden jars charged by it may be .discharged in at least three rather distinct ways. The first is by simple metallic con-

duction between the charged bodies, as when the electrodes are united by a wire. The only phenomenon connected with such a discharge is the generation of heat in the conductor which is strictly according to the I^2R law of dynamic electricity. The current, however, is not uniform and a more adequate expression for the heat generated — assuming all the energy of the discharge is transformed into heat — is

$$H = \frac{Q\ V}{2\ J} \text{calories},$$

where J is the mechanical equivalent heat, expressed in ergs per calorie. ($J = 42 \times 10^6$.) For it was shown in section 34 that $\frac{1}{2}\ Q\ V$ represents the total energy of a charge. The energy thus set free is usually too insignificant to be detected, but discharges from large machines equipped with heavy condensers have been made to heat a considerable length of wire, and to fuse pieces of tinfoil.

In elaboration of an experiment by D'Arsonval, Dr. Sheldon has succeeded in lighting to full brilliancy an ordinary 16 candle-power 116 volt incandescent lamp. He connected the electrodes of a large Holtz machine respectively with the inner coatings of two Leyden jars. The electrodes were set with a reasonably large spark gap. The outer coatings of the jars were connected together through an impedance coil consisting of half a dozen turns of heavy copper wire around a four-inch glass cylinder. From the extremities of the impedance coil wires were led off to the lamp. After suitable adjustment of spark gap and speed the lamp glowed at full brilliancy. A hot-wire ammeter inserted in the lamp circuit showed the lamp to be taking its normal current.

The second method of discharge is the so-called " dis-

ruptive discharge," meaning a discharge through the agency of a spark. If an influence machine be operated without Leyden or other condensers it will be found impossible to cause the discharge to cross anything but a small gap. When the electrodes are brought sufficiently near together, say within an inch, a stream of fine sparks keeps up with such rapidity of succession as to look like one continuous thread of light between the terminals. This thread does not, however, stay stationary, except in the case of particularly short gaps, but cuts out an ever-changing zigzag course from one electrode to the other. Such a discharge is accompanied by a gentle hissing noise. If now a couple of Leydens be added, their inside coatings connected to the electrodes respectively and their outer coatings connected together, the discharge loses its continuous character and becomes intermittent. At regular intervals of a few seconds what appears to be a single bright spark passes accompanied by a sound like the snap of a whip. If now the electrodes be separated the intervals between sparks become longer and the discharge becomes brighter and more noisy. The more jars that are added, and, up to the sparking limit of the machine, the wider the separation of the electrodes, the more crashing the reports become. As the spark becomes longer it appears to take on a forked shape, and a photograph of such a spark usually shows many forkings. The branches of such a fork invariably point toward the negative electrode, or better, more nearly to the negative than the positive. Faraday showed by experiment that when the electrodes were tipped with spheres of unequal diameters, the longest sparks were obtainable when the smaller sphere was made the positive electrode.

The disruptive strength of such a spark is considerable. A thick book can be held between the electrodes and a hole will be pierced through it. So also can a hole be pierced through glass or mica, though the thickness of these must not be great and the electrodes must be brought close to the surface to be punctured. Frequently the puncture cannot be made unless the electrodes are made of sharp points instead of the usual spheres. These points then tend to concentrate the stress that might otherwise be spread over a considerable surface, the positive electricity spreading out on one side, the negative on the other, the glass or mica functioning merely as the dielectric of a condenser. The insulator must also be of sufficient size, for the spark will travel a long way over the surface to get around the edge rather than pierce the substance. The distance the spark will travel in this manner is greatly in excess of the distance it would jump in open air because of the conductive properties of the surface, which must of necessity be somewhat moist and dirty.

If such a discharge be passed through a piece of paper or cardboard, a curious effect can be observed. It will be found that the hole is not only burred on one side as it would be if it had been pierced with a needle, but is burred on both sides. This has been taken as an indication of the oscillatory character of the discharge, but the same effect would be observed if the stresses in the air are such as to pull at the card from each side.

Prof. S. P. Thompson gives the following formula for approximately determining the voltage necessary to start a spark across an air gap of l centimeters, the gap in any event being greater than two millimeters.

$$V = 1{,}500 + 30{,}000\, l.$$

If the length of the gap is expressed as L inches, the same formula becomes

$$V = 1{,}500 + 76{,}000\ L.$$

The third method of discharge is by convection. This form of discharge can take place in any non-conducting fluid, either liquid or gaseous, but the only form of importance is the discharge in air. If an influence machine be well excited and the terminals separated to greater than the sparking distance, a discharge will take place from each terminal which can be seen in a dark room as a light blue luminescence branching out in such shape as to give it the name of *brush discharge*. A hissing noise is always heard in connection with this phenomenon. The electric wind as described in section 14 is the simplest form of convection discharge, but this can only emanate from a sharp or pointed conductor. If, as in the case above described, the conductor is finished entirely in curves and spheres, such a starting point is lacking and the charge seems to take the form of a continuous spark for a short distance from the conductor. This spark branches and radiates into numberless paths, each of which becomes a moderately good conductor and conveys the charge to a point where it can be transferred to the air particles as in the electric wind. A difference can be observed in the brush discharges from the positive and from the negative poles, the brush at the positive pole being larger and more bushy than the other.

Spectrum analysis of the brush discharge shows the presence of volatilized metal of which the conductor is composed. In the disruptive discharge if the terminals be of different metals, traces of both metals can be found in the spark. This indicates the actual consumption of the metal

of the electrodes and accounts for the pitting observed on the surface of the discharging knobs after they have been in service some time. A pair of knobs which when smooth and well polished will give a disruptive discharge across a certain gap will often give a brush discharge when spaced the same but with a surface roughened and pitted. Therefore, to get the maximum length of spark from a machine the electrodes should be kept in a bright condition.

It is found that the resistance of air to the passage of electricity decreases as the pressure decreases until a certain point is reached, after which it increases with the decrease of pressure till at the nearest approach to a vacuum that can be practically attained the resistance is much higher than at atmospheric pressure. The discharges which occur through tubes exhausted to these vacuums are believed to be due to convection. They give rise to many interesting and unexplained phenomena, Geissler effects, cathode rays, Roentgen rays, Becquerel rays and others, a discussion of which is entirely without the scope of this present work.

APPENDIX.

A. Dimensions of Units.— Nearly, if not quite all, the quantities dealt with in exact science can be expressed in terms of three fundamental units. The choice of these three is to a great extent a matter of convenience, for there are many combinations of units that might be employed. For instance, all units could be referred to a fundamental unit of mass, one of energy, and one of density; or to one of mass, one of velocity, and one of time. The units actually chosen for use in the *c. g. s.* system, which is the only system that has any pretension to being uniform and international, are as follows:

A definite length, called a *centimeter,*
A definite mass, called a *gram,* and
A definite interval of time, called a *second.*

These units were chosen chiefly because they admitted of accurate and relatively easy comparison with other quantities of the same kind; because permanent standards of them could be made (except of time. This depends upon the uniformity of the earth's rotation on its axis); and because the standards do not vary with gravitation nor barometric pressure.

These three units are called the *fundamental units.* All others are called *derived units.* All units based directly upon the fundamental units, without the intervention of reduction factors, are called *absolute units.* The simplest of the derived units is that of area. We could of course assume some unit for this, the acre for instance, but this would interfere with the uniformity of the system. We

therefore consider an area as a length times a length. Letting A, L', and L'' stand respectively for the area, one length and the other length,

$$A = L' L''.$$

To get the unit area according to this system we must let L' and L'' each equal unity, dispensing with the need of the indices. Then

$$A = LL = \mathrm{L}^2$$

and we say that the dimension of the unit of area is L^2. The dimension of the unit of volume is an area times a length,

$$V = \mathrm{L}^2 \mathrm{L} = \mathrm{L}^3$$

and we say the dimension of the unit of volume is L^3.

A velocity, whether expressed as miles per hour, feet per minute, centimeters per second, or otherwise, cannot be considered as anything but a certain distance covered in a certain time unless we fall back on other than our fundamental units. (An *acceleration* times a time would give a velocity.) We therefore say that the dimension of the unit of velocity is a distance divided by a time, or

$$v = \mathrm{LT}^{-1}$$

Acceleration is defined as the increase in velocity per unit of time. Hence the dimension of the unit of acceleration is a velocity divided by a time, thus

$$a = v\,\mathrm{T}^{-1} = \mathrm{LT}^{-2}$$

The unit of force is defined as that force which acting on the unit of mass for a unit of time generates in it a unit of velocity. Evidently the force varies directly as the mass, directly as the velocity, and inversely as the time. The dimension of this unit will then have a mass and a velocity in the numerator and a time in the denominator, or

$$f = v\,\mathrm{MT}^{-1} = \mathrm{MLT}^{-2}$$

According to another definition the unit of force is that force which will impart a unit acceleration to a unit mass. According to this definition the dimension of the unit is

$$f = a\,\mathrm{M} = \mathrm{MLT}^{-2}$$

which is the same as before.

So with any unit, not only these simpler mechanical ones, but also with the more complex magnetic and electrical ones; by referring to its definition, the dimension of any unit can be written in terms of L, M, and T, or in terms of other units which in their turn have dimensions consisting solely of L, M, and T.

In Table A, which is in part adapted from S. P. Thompson's Electricity and Magnetism, is given in convenient form for reference, data concerning the various units used in the study of electricity. The dimensions of the electrostatic, the magnetic, and the electromagnetic, called also the electrodynamic, systems of units are given, together with short definitions of each sufficient to show how its dimension is derived from one or more units whose dimensions have previously been given. Where the absolute units have corresponding practical units, the names of the latter together with their ratio to the absolute or *c. g. s.* values are given.

A few examples of the less obvious derivations of dimensions will now be given. For instance, to determine the dimensions of the electrostatic unit of quantity of electricity. It was shown in section 12 that the force between any two quantities varied directly as their product and inversely as the square of the distance between them. That is, if proper units be taken,

$$f = \frac{q'\,q''}{l^2}.$$

TABLE A.

Usual symbol.	Unit.	Definition.	Dimension.	Practical unit.	Practical c. g. s.
		General.			
l	Length (centimeter)	Fundamental	L		
m	Mass (gramme)	"	M		
t	Time (second)	"	T		
A	Area (sq. cm.)	Length × length	L^2		
v	Velocity	Length ÷ time	LT^{-1}		
a	Acceleration	Velocity ÷ time	LT^{2}		
f	Force (dyne)	Mass × acceleration	MLT^{-2}		
w	Work erg)	Force × length	$ML^{2}T^{-2}$		
		Electrostatic.			
Q	Quantity*	$\sqrt{\text{Force} \times (\text{distance})^2}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$		
V	Potential	Work ÷ quantity	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$		
C	Capacity	Quantity ÷ potential	L		
F	Intensity	Force ÷ quantity	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}$		
	Current†	Quantity ÷ time	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$		
	Resistance†	Potential ÷ current	$L^{-1}T$		

Magnetic.

m	Strength of pole* / Quantity magnetism.	$\sqrt{\text{Force} \times (\text{distance})^2}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	.....	.....
V M.M.F.	Magnetic potential / Magnetomotive force	Work ÷ strength of pole	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	.....	.....
H	Intensity of field,	Force ÷ strength of pole	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$	ampere-turn per cm.	$\frac{10}{4\pi}$
$\mathcal{B}$	Magnetic density ‡	Force ÷ strength of pole	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$	Gauss	1
φ	Magnetic flux	Mag. density × area	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	Maxwell.	1
R	Reluctance	Mag. flux ÷ M.M.F.	L	.....	.....
$\mathcal{P}$	Permanence	1 ÷ reluctance	L^{-1}	.....	.....

Electromagnetic.

V E.M.F.	Potential / Electromotive force	Magnetic flux ÷ time	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	Volt	10^{8}
Q	Quantity	Potential ÷ work	$M^{\frac{1}{2}}L^{\frac{1}{2}}$	Coulomb	10^{1}
I	Current	Quantity ÷ time	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	Ampere	10^{-1}
R	Resistance	E.M.F. ÷ current	LT^{-1}	Ohm	10^{9}
C	Capacity	Quantity ÷ potential	$L^{-1}T^{2}$	Farad	10^{-9}
P	Power	Current × E.M.F.	$ML^{2}T^{-3}$	Watt	10^{7}
L M	Self-induction / Mutual induction	Mag. flux ÷ current	L	Henry	10^{9}

* In air. † Not used. ‡ $\mathcal{B}=\mu H$, and in air $\mu=1$.

To get unit conditions the two quantities must be equal and their product can be written Q^2, then

$$Q^2 = fl^2.$$

Now the dimension of a force is MLT^{-2} and of a length is L. So we may write for the dimension of the electrostatic unit of quantity

$$Q = \sqrt{fl^2} = M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \times L = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}.$$

An analogous line of reasoning may be followed in getting the dimension of the unit magnet pole.

The electrostatic potential at any point varies directly as the work required to bring a quantity of electricity to that point from an infinite distance and inversely as that quantity. So we can write

$$V = \frac{w}{Q} = M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}.$$

The difference in electromagnetic potential between the ends of a conductor moving through a magnetic field varies directly as the number of lines of force cut and inversely as the time taken to cut them. The number of lines of force, or the magnetic flux, has the dimension $M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$ hence the unit difference of electromagnetic potential, called also potential and electromotive force, has the dimension $M^{\frac{1}{2}} L^{\frac{2}{2}} T^{-2}$.

B. Ratio of the Units.— It can be observed from Table A that the dimensions of similar units in the electrostatic and the electromagnetic systems are different. Thus the electrostatic unit of quantity has the dimension $M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$ while the electromagnetic unit has the dimension $M^{\frac{1}{2}} L^{\frac{1}{2}}$ This difference, which exists throughout the two series arises from faulty definitions of the electrostatic unit of

quantity in the one case and the magnetic unit of quantity (strength of pole) in the other. Every electrostatic field is set up in some medium that has a dielectric constant k (called also specific inductive capacity) and every magnetic field is set up in some medium that has a permeability μ. Neither the dimension nor the absolute value of either of these factors is known, and the values of k and μ in air are taken as unity in lieu of better units. If the dimensions

TABLE B.

Name of Unit.	Dimensions.			Ratio of Units.
	Electrostatic.	Electromagnetic.	Electrostatic. / Electromagnetic.	c.g.s. static / Pract. magnetic.
Quantity..	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$	$M^{\frac{1}{2}} L^{\frac{1}{2}}$	$L T^{1} = v$	$\frac{1}{3} \times 10^{-9}$
Potential..	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}$	$L^{-1} T = v^{-1}$	300
Current...	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$	$L T^{-1} = v$	$\frac{1}{3} \times 10^{-9}$
Resistance.	$L^{-1}T$	$L T^{-1}$	$L^{-2} T^{2} = v^{-2}$	9×10^{11}
Capacity...	L	$L^{-1} T^{-2}$	$L^{2} T^{-2} = v^{2}$	$\frac{1}{9} \times 10^{-11}$

Note.— The last entry is for farads. For microfarads, $\frac{\text{c. g. s. static}}{\text{microfarads}} = \frac{1}{900{,}000}$

and absolute value of these factors were known, and they were properly introduced into the definitions of their respective systems, the dimensions and values of the corresponding terms of the two sytems would be in accord.

Furthermore it is irrational that the dimension of a unit should contain such terms as $M^{\frac{1}{2}}$ or $L^{\frac{3}{2}}$ since these terms have no physical meaning. Rücker has shown that the dimensions of the units could be rationalized and the

two systems brought into accord if k and μ were properly introduced into the definitions and if their product were assumed to have the dimension of the reciprocal of the square of a velocity.

In Table B are given the ratios of the dimensions of similar units in the two systems. It may be noted that in these ratios only the terms L and T appear and that these invariably appear with equal and opposite indices. That is, the ratio in every case is some power of the dimension of a velocity, which may be represented by v. This is a definite concrete velocity and its value is this: two parallel wires carrying currents in the same direction are mutually attracted. An electrostatic charge being moved through space produces to a certain extent the effect of a current of electricity flowing along the path of its motion. The extent of this effect depends upon the velocity with which the charge is moved. Two similar electrostatic charges are mutually repelled. The velocity v which enters into the ratios of the dimensions of the two systems is that velocity at which two quantities of electricity should travel side by side so that their mutual electromagnetic attraction will be just equal to their mutual electrostatic repulsion.

The numerical value of v in centimeters per second has been experimentally determined by several investigators.

Weber and Kohlrausch compared the two units of quantity and gave

$$v = 3.1074 \times 10^{10}.$$

Kelvin compared the two units of potential and found

$$v = 2.825 \times 10^{10}.$$

Clerk Maxwell balanced an electrostatic attraction against an electrodynamic repulsion and gave

$$v = 2.88 \times 10^{10}.$$

Ayrton and Perry, by measuring the capacity of an air condenser electrodynamically and determining its electrostatic capacity from consideration of its dimensions found

$$v = 2.980 \times 10^{10}.$$

These values all agree very closely with the velocity of light in vacuo, which is approximately 3×10^{10} centimeters per second. This is a most important fact in support of the electromagnetic theory of light.

In Table B are also given the ratios of the *c. g. s.* electrostatic units to the practical electromagnetic units, assuming $v = 3 \times 10^{10}$. In deriving these ratios it will be noticed that the ratio of the absolute units is the *inverse* ratio of their dimensions. This at first glance may seem paradoxical, but the following explanation of the case of the units of quantity may serve to make this matter clear.

The dimension of the static unit of quantity is $M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$, that of the dynamic unit $M^{\frac{1}{2}} L^{\frac{1}{2}}$. It is therefore evident that some system of units of length, mass, and time can be chosen so that the actual value of the two units of quantity will be the same notwithstanding the difference in the naming of their dimensions. In this new system then let the

unit of length equal L centimeters, the
unit of mass equal M grams, and the
unit of time equal T seconds.

The static unit of quantity in this new system will contain $M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$ of the *c. g. s.* static units of quantity, and likewise the dynamic unit will contain $M^{\frac{1}{2}} L^{\frac{1}{2}}$ of the *c. g. s.* dynamic units. But by our preliminary assumption these

two unit quantities in the new system are equal. Therefore

$$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} \times \text{one } c.\ g.\ s. \text{ static unit} = M^{\frac{1}{2}} L^{\frac{1}{2}} \times \text{one } c.\ g.\ s. \text{ dynamic unit.}$$

or

$$\frac{\text{one } c.\ g.\ s. \text{ static unit}}{\text{one } c.\ g.\ s. \text{ dynamic unit}} = \frac{M^{\frac{1}{2}} L^{\frac{1}{2}}}{M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}} = L^{-1} T = \frac{1}{v}.$$

Now v is a definite concrete velocity, entirely independent of the size of the fundamental units. In the new system which we assumed above v is evidently that velocity which would be designated by unity. In the *c. g. s.* system v is found by experiment to be 3×10^{10}. In any system whatsoever the ratio

$$\frac{\text{electrostatic unit of quantity}}{\text{electrodynamic unit of quantity}} = \frac{1}{v}$$

holds true. This, it is now observed, is the reciprocal of the ratio of the dimensions of the quantities.

In a like manner it can be shown for each of the other pairs of units in the two systems, that the ratio of the units is the inverse ratio of their dimensions.

The above reasoning is adapted from Professor J. D. Everett's "Units and Physical Constants," to which work the student is referred for further information as to the values and relations of the *c. g. s.* units.

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